

# A Model of Partial Regulation in the Maritime Ferry Industry\*

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*Preliminary Version*

## Abstract

We investigate the peculiar features of the maritime ferry market and characterize the optimal regulatory policy for the industry, referring to environments where shipping is the unique remedy to remoteness. When the original monopoly is opened up to competition, the *optimal partial regulatory policy* reflects the trade-off between high-season inefficiency and low-season equity, which follows from demand seasonality. We show that, under this policy, continuative provision of the service is secured by the incumbent, but the rivals' incentives to operate during the low season are dampened. Moreover, the need to ensure budget balance for the regulated shipper softens competition in the high season and makes net rents available to the unregulated operators. We as well highlight how the optimal policy pins down the desirable subsidization across passengers and their allocation between operators. We find that, all over the year, the islanders are allotted to the dominant enterprise and that their consumption is let be partly financed by the non-residents, who eventually switch to the competitors. We lastly establish how the optimal partial regulatory policy should be implemented. We demonstrate that decentralization of the target allocation is to be based on data about the overall traffic. This result, which easily extends to other liberalized sectors, suggests how to apply the price-cap method to partially regulated environments, as opposed to traditional monopolies.

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## EXECUTIVE SUMMARY

In this paper, we first investigate the peculiar characteristics of the maritime ferry market. With reference to environments where shipping is the unique remedy to the remoteness of the islands from the mainland, we then characterize the optimal regulatory policy for the industry in the recently liberalized European context.

The demand for shipping services is characterized by strong seasonality. During the low season, the traffic volume can be so little that ferry service provision becomes unattractive to profit-maximising operators, hence either it does not take place at all, which undermines the territorial cohesion and socioeconomic integrity, or captive segments are inequitably overcharged for the service. On the opposite, during the high season, significant potential rents induce providers to be active and the traffic is served, yet market power is exerted (especially) on the less elastic demand segments. Public intervention is then called upon for socially desirable allocations to entail in the different seasons and market segments, at the minimum reliance on costly public funds.

For years the institutional setup has not convincingly matched the shipping sector peculiarities, often leading to resource waste through soft budgets and abusively diluted subsidies. Nevertheless the issue of how to regulate the industry has received scarce attention from the economic literature. Researchers have generally concentrated on more "visible" sectors. Relevant such studies are the one by Billette de Villemeur (2004) about airlines and those by Cremer *et Alii* (1997, 2001 and 2002) about postal services. Though non-profitability of remote areas and exercise of market power on inelastic demand segments concern all the mentioned industries, regulation of the maritime transport requires specific analysis, especially in the light of the social aspects associated with the territorial continuity.

For the study to be properly performed, it is crucial to consider that the configuration of the European maritime ferry sector is currently evolving. The EU Regulation 3577/92 extended the service freedom principle to cabotage and short-hauls connections as from 1999. As a result, access by new unregulated competitors to former regulated monopolies is registered in several European countries. This evolution toward *partially regulated oligopolies* generates regulatory trade-offs, which were not previously present. In particular, it becomes necessary to reconcile the contrasting interests of a wider range of economic agents: the dominant firm has to survive under the competitive pressure exerted by the upcoming operators, while passengers pursue service affordability and high frequency. This process has been investigated for sectors such as the telecommunications (Biglaiser and Ma, 1995), but again it lacks comprehensive economic foundation as far as the shipping activity is concerned, despite the non-negligible amount of resources involved.

Hinging on the considerations above, in the present paper we envisage a European-like shipping framework and characterise the optimal partial regulatory policy. This policy reflects the trade-off emerging from the seasonality issue previously mentioned, which is ultimately a trade-off between high-season inefficiency (some consumer rationing and large potential profits) and low-season equity (service provision and price affordability for the islanders).

Our investigation returns a few interesting predictions about the optimal policy as well as useful insights about its implementation. In particular, our model predicts that burdening the incumbent with regulatory obligations secures continuative service provision, but dampens the rivals' incentives to operate during the low season. In turn, the need to ensure budget balance for the regulated shipper softens competition in the high season and makes net rents available to the strategic unregulated operators. Importantly, under these circumstances, average market price may prove higher, but travel scheduling more

intense in partially regulated oligopolies than in regulated monopolies.

We as well show how the optimal policy pins down the desirable subsidization across passengers and their allocation between operators. We find that, all over the year, the islanders are allotted to the dominant enterprise, which is compelled to provide sufficiently favourable conditions. Their consumption is let be partly financed by the non-residents, who essentially travel in the high season. These passengers have an incentive to switch to the competitors, when the latter offer them cheaper services.

As a final step, we establish how the optimal partial regulatory policy should be implemented. We demonstrate that decentralization of the target allocation to the regulated provider is to be based on aggregate data about the *overall* traffic in the market, a conclusion which would easily extend from the shipping industry to other liberalized sectors. Interestingly enough, this result suggests how to apply the widespread price-cap method to partially regulated environments, as opposed to traditional monopolies.

# 1 Introduction

Democratic Constitutions recognize individual mobility (broadly intended) as a human fundamental right. For instance, Art. 16 of the Italian Constitution states: "Every citizen can circulate (...) in any part of the national territory (...)" ; this freedom is viewed as a means to the individual full development and effective participation in the Country's organization, both of those promoted by the Italian Republic (Art. 1).

Hinging on the generalized constitutional recognition, the *universal service principle*, which translates into the *territorial continuity principle* as far as mobility is concerned, is called upon for the purpose of limiting the geographic impediments and the resulting socioeconomic difficulties, which penalize the people leaving on the islands. This amounts to ensuring that the islanders are connected to the continental territory in ways as close as possible to the mainland inhabitants at affordable charges.

Maritime transportation critically contributes to secure the national cohesion and integrity, hence it is perceived to be a service of general interest. Administrative prescriptions for service provision have traditionally stemmed from this circumstance, lacking any convincing theoretical background. For several years, ferry companies have operated as monopolists, eventually entitled with exclusive rights to serve specific geographical areas. Public undertakings have been entrusted with operation in countries such as Italy, Spain and France. In general, long-term concession contracts (20 to 25 years) have been awarded without public tendering procedures, either in consideration of the public nature of the company or because, at the time, there was no European norm on the matter. A good example is given by the contract for the provision of ferry services between Corsica and France, which was inaugurated in 1976 and is still in place.

At the European level, the configuration of the maritime ferry sector is destined to evolve in the close future. Many of the long-term contracts mentioned above are approaching expiration. Some of the publicly owned companies are supposed to be privatised in the short run. To some extent, entry of unregulated shippers is currently registered, after the service freedom principle has been extended to cabotage and short-hauls connections by the EU Regulation 3577/92 [17]. Yet even the guidelines through which the European Commission has attempted to discipline the (transition to the) ultimate organization of the industry do not rest on a comprehensive economic foundation.

In the present paper, we address the issue of how appropriate institutional settings should be designed for the operation of maritime ferry services in environments where no substitute transportation mode is available. We first take a pure efficiency perspective, which corresponds to the case where social welfare coincides with total surplus. We subsequently concentrate on the *public service obligations* (PSOs) which ought to be imposed "upon a carrier to ensure the provision of service satisfying fixed standards of continuity, regularity, capacity and pricing (...)" (EC [16]) and discuss the far-reaching distributional implications that are associated.

So far, this subject has received incredibly little attention even from the specialized literature. The lack of interest might have been justified by the relatively small size of the industry, as compared to other transport sectors. Researchers have generally believed that it was enough to study air transportation to know all that matters about maritime transportation. In the same spirit, practitioners have typically regarded maritime transportation as a minor substitute for air transportation. An example of this attitude can be found again in the contract for Corsica; in the latter, reductions in ferry tariffs for residents are fixed far more restrictive as compared to air transport reductions<sup>1</sup>. In our view, this approach is unsatisfactory; instead, specific analysis is required in the light of the distinctive features of the service, namely the strong seasonality of demand, the tech-

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<sup>1</sup>More precisely, the principle of residential tariffs for ferry services is made conditional on the event that the islanders transfer also their cars.

nological characteristics of service provision and the social aspects of territorial continuity. Moreover, given the amount of resources involved, it is misleading to affirm that the shipping market be negligible with respect to the economy of the various countries and of the EU as a whole.

In our work, we focus attention on the institutional design of the sole industry structures which are destined to be relevant, given the way the sector is likely to evolve in the European context, namely monopoly and duopoly.

Monopolies, whether public or private, survive in scenarios where the cabotage liberalization process has no impact on the industry structure. Whenever this is the case, the level of competition remains negligible and cannot be reasonably expected to improve soon. In such a perspective, our analysis shares the same spirit as the one performed by Billette de Villemeur [3]. Indeed, the latter focuses on situations of similar kind, which materialize in the air transportation sector, despite the 1997 liberalization.

On the opposite, oligopolies (are destined to) realize in the event that partial deregulation does induce access by additional operators. If entry occurs, then the new shippers play the market game as Stackelberg followers *vis-à-vis* the regulated incumbents, hence *vis-à-vis* the regulatory authority. Again this is not a peculiarity of the maritime ferry industry. Biglaiser and Ma [2] refer to the long-distance telephone segment in the telecommunication sector as an example of analogous phenomena appearing in the other utilities that have recently been opened up to competition.

In a complete-information environment, we characterize the optimal monopoly regulation as well as the optimal duopoly partial regulation. More precisely, we determine which *prices* the compelled shipper should charge and how many *connections* it should operate for the social objectives to be achieved. From the firm's standpoint, these constitute duties which, to rephrase the EU Regulation 3577/92 [17], would not be assumed, as long as pure commercial interests were to prevail. As far as the duopoly is concerned, this as well involves fixing the second operator's price and frequency, given its market strategy. In this sense, the efficient "public-private" combination of maritime service provisions remains characterized, "public" meaning "controlled by the public authority".

A substantial point is that the optimal regulatory policy crucially follows from the goals it is meant to pursue. Indeed, targeting pure efficiency is not the same as targeting redistribution aims. In particular, the price-and-frequency bundle, which represents the most (constrained) efficient performance of the industry for a utilitarian society, does not need to correspond to the one which secures a reasonable level of territorial continuity.

To stress this difference, we first characterize the regulatory policy which is pinned down when society has a utilitarian attitude and is essentially concerned that the market equilibrium be as efficient as feasible. We as well argue that it can be implemented by imposing a properly structured constraint to the regulated shipper, in which the relevant decision variables are combined to provide desirable incentives.

At later stage, we highlight that the regulatory solution might need be amended for equity considerations to be incorporated in favour of the people who are penalized by the drawbacks of insularity. We find that, in order to fund the costs of the territorial continuity system, it might be necessary to require the non-residents to provide implicit subsidies for the islanders' consumption of ferry services. We also demonstrate that, under duopoly, such subsidies can be somewhat escaped by patronizing the entrant. Yet any advantage associated with the presence of the unregulated shipper comes at the price of letting it pocket a net rent.

The regulated operator never obtains positive benefits, as long as the optimal regulation is implemented, whatever the social objectives. Nevertheless, in all regimes, we require that its budget constraint be met. This might appear in contrast with the circumstance that, in Europe, transfers have traditionally been and are still feasible. In fact, it is less so than one would perceive at a first glance.

To some extent, the diffusion of public shippers explains the long-lasting history of *ex post* diluted (direct and indirect) subsidies. Indeed, as Martimort [21] underlines, when the State owns a firm, it is likely unable to refrain from using public funds to transfer resources in favour of the firm. A good example is given by the Italian shipping industry. Bergantino [1] reports that a substantial part of the Italian ferry traffic is subsidized and the yearly expense for the public budget is ultimately close to 250 million euros. Nevertheless, the list of countries concerned by the subvention practice also includes those where private shippers are active; the resulting bill is not less significant. In the UK, where only some of the lines off the Scottish coast are subsidized, the associated cost exceeds 50 million euros per year. Greece, Ireland, Portugal, Germany, Denmark, Finland, all have subsidized ferry services (see European Commission [15]). This suggests that the subvention habit hinges also on considerations other than the ownership structure, namely the necessity to secure that the service be provided on lines, in areas and in periods that are not self-financing.

The European Commission has recently intervened to remove the *abusive* aspect of the tradition. For this purpose, it has ruled in the direction of containing the amount of aids Member States can provide to maritime transport (see European Commission [16] and [13]). On one side, this should prevent too generous an attitude toward public shippers and might possibly accelerate privatization in countries which have strong laws against budget deficits and restrictions on borrowing<sup>2</sup>. On the other side, it is meant to preserve justifiable supporting measures. Indeed, according to the current norms, subsidies can be granted to compensate for public service obligations; furthermore, operators involved in public service contracts (PSCs) are entitled to be refunded the extra costs incurred by supplying the service, provided that the reimbursement is "directly related to the calculated deficit" (EC [16]).

In analytical terms, satisfying the operator's budget constraint encompasses both environments where transfers from the government are not allowed and environments where the regulated firms can be awarded subventions. Indeed, with the opportune qualifications, the solution is formally (though not numerically) equivalent. Therefore, the budget-balance modelling device has the advantage of remaining neutral with respect to the subsidy/non-subsidy option, while better representing the European conservative attitude.

The paper is organized as follows. In Section 2, we present the model. We first develop a detailed description of passengers' preferences and behaviour; we subsequently illustrate the supply side of the market by focusing on the shippers' technologies and profit functions. In Section 3, we characterize the utilitarian first-best benchmark. In Section 4, we assume that society pursues efficiency objectives and determine the optimal monopoly regulation accordingly; we then illustrate how it can be decentralized. In Section 5, after assessing the impact of the incumbent's actions on the entrant's decisions, we characterize the optimal partial regulation and explain how it should be decentralized. Step by step, the duopoly results are paralleled to the monopoly ones. Section 6 is devoted to addressing the redistribution concerns of society. The implications of applying the territorial continuity principle are discussed. Section 7 concludes.

## 2 The Model

We consider a domestic ferry industry, which provides maritime transportation services connecting localities that are separated by the sea, such as the islands and the continental territory of a country.

In our stylized market, travellers are assumed to be heterogeneous, the source of het-

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<sup>2</sup>Again, see Martimort [21], who argues that it is generally easier to enforce laws which prevent regulators from providing *ex post* transfers to the regulated firms rather than laws which interdict Treasury manipulations. Therefore, the State can more credibly commit to hard budget constraints as the regulator of a firm rather than as its owner.

erogeneity being twofold. Indeed, each individual is characterized by a taste parameter  $\alpha$  and exhibits a time value  $\tau$  expressing the opportunity cost of the time spent waiting for a connection. Both  $\alpha$  and  $\tau$  are assumed to take values on the compact interval  $[0, +\infty)$ ; we let  $g(\alpha, \tau)$  denote their joint density function. Furthermore, the population of passengers classify into two essential categories, namely the residents of the islands (market segment  $r$ ) and the non-residents (market segment  $n$ ).

We initially concentrate on the case where a dominant firm (enterprise  $I$ ) operates as a regulated monopolist. We subsequently focus on the scenario where a (potential) competitor (enterprise  $E$ ) accesses the industry and supplies its service as an unregulated Stackelberg follower, whereas shipper  $I$  acts as a regulated leader.

The basic period of operation is considered to be the year; nevertheless, to capture the significant seasonality of the industry activities, we identify two main seasons, which we denote by  $s = l, h$ , where  $l$  stays for low season and  $h$  for high season.

The service is characterized by both a monetary and a quality dimension, which constitute the relevant choice variables in the industry. The monetary dimension is given by the price that is charged by the operator supplying the service; each category of passengers can be offered a different price in each season<sup>3</sup>. On the other hand, the quality dimension consists in the number of performed travels, which is allowed to vary on a seasonal basis. Once both dimensions are accounted for, the transportation services provided in duopoly can be viewed as perfect substitute products. Stemming on the substitution property, we suppose that passengers may behave in either of the ways described in the following Section.

## 2.1 The Preferences and Demands

We hereafter illustrate how people make their travel decisions, hence how the demand for the transportation service is formed. Though we perform the investigation for the shipping market specifically, it is clear that it might be extended to alternative contexts, namely bus, train and air transportation.

We initially adopt the perspective of the single traveller. We subsequently use the results achieved at the individual level to derive the relevant aggregate functions. For the time being, the classification of travellers into residents and non-residents is irrelevant and so neglected. It will matter as soon as the firms' and the regulator's standpoints are introduced into the picture, hence we will come back to it at that stage.

For sake of shortness, we content ourselves with studying passenger behaviour for the case where two shippers are active in the industry; instead, we renounce to detail over the monopoly situation. As it will become rapidly evident, the latter should simply be viewed as a special, much simpler case of the scenario we focus on.

Some travellers fully exploit the option of screening the more suitable market proposal. This involves that they select the operator whose price-and-frequency policy makes them better off and choose the number of tickets to purchase from it. Reasonably enough, these customers exhibit regular and recurring travelling necessities; for instance, they need to daily reach their working place. Hence, they are able to systematically plan their movements. For simplicity, we say that these are the passengers of type 1.

The remaining customers (hereafter, type-2 passengers) take advantage of the first available connection, indifferently of the price they need to pay for the ticket and whatever the operating firm. One can imagine that these passengers mainly travel for occasional reasons, such as touristic visits. To their impatience they sacrifice the option of choosing between operators. As a result of this attitude, they perceive the transportation service as a unique good, as if they were faced with an "aggregate monopoly", albeit they actually randomize over the two services.

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<sup>3</sup>Price discrimination is a common practice in transportation industries (see, for instance, Wilson [29]).

Hinging on the behavioural features previously illustrated and assuming that all relevant costs and benefits are correctly anticipated and incorporated into the personal programmes, we can write the net utility (surplus) function of either type of traveller. In particular, for a type-1 customer exhibiting taste parameter  $\alpha$  and time value  $\tau$ , we have

$$S(\alpha, \tau; x_j^{s,1}) = \sum_s \left[ \alpha U(x_j^{s,1}) - \left( p_j^s + \frac{\tau}{2f_j^s} \right) x_j^{s,1} \right]. \quad (1a)$$

In (1a),  $\alpha U(\cdot)$  is the gross utility function, increasing and concave in the argument  $x_j^{s,1}$ ; the latter represents the number of tickets the  $(\alpha, \tau)$ -individual buys from the selected firm  $j$  in season  $s$ . Furthermore,  $p_j^s$  is the tariff charged and  $f_j^s$  the number of connections supplied by operator  $j$  in season  $s$ . The sum  $(p_j^s + \tau/2f_j^s)$  measures the so-called *generalised cost*, which is given by the monetary price together with the disutility associated with the departure delay  $(\tau/2f_j^s)$ ; hence, it is the total unit cost the passenger bears. In particular, the ratio  $1/2f_j^s$  is determined under the hypothesis that the ideal departure time is uniformly distributed along the time interval between any two departures<sup>4</sup>. The functional form in (1a) is inherited from Billette de Villemeur [3], who adopts it in a model of air transport monopoly regulation; nevertheless, the present framework is richer than his, as both seasons and customer types are allowed for.

The surplus function of the type-2  $(\alpha, \tau)$ -traveller is a modification of the previous one; it is given by

$$S(\alpha, \tau; x^{s,2}) = \sum_s \left[ \alpha U(x^{s,2}) - \left( p^{s,e} + \frac{\tau}{2f^s} \right) x^{s,2} \right], \quad (1b)$$

where  $x^{s,2}$  expresses the total number of tickets he buys in season  $s$  (from whatever firm) and  $f_E^s = (f_j^s + f_k^s)$  the total amount of connections offered by the industry in the same season<sup>5</sup>. Furthermore,  $p^{s,e} = (f_j^s p_j^s + f_k^s p_k^s) / f^s$  indicates the price the customer expects to pay, which is perceived to be a weighed sum of the tariffs  $p_j^s$  and  $p_k^s$ , weights being the relative frequencies  $f_j^s/f^s$  and  $f_k^s/f^s$  respectively. It follows that the generalised cost  $(p^{s,e} + \tau/2f^s)$  is now represented by the sum of the perceived price and the disutility associated with the departure delay<sup>6</sup>.

The optimal type-1 demand for travels  $x_j^{s,1}(\alpha, \tau; p_j^s, f_j^s)$  is characterized by the condition

$$\alpha U'(x_j^{s,1}(\alpha, \tau; p_j^s, f_j^s)) = p_j^s + \frac{\tau}{2f_j^s}, \quad \forall s, \quad (2a)$$

while the type-2 demand  $x^{s,2}(\alpha, \tau; p^{s,e}, f^s)$  is determined by

$$\alpha U'(x^{s,2}(\alpha, \tau; p^{s,e}, f^s)) = p^{s,e} + \frac{\tau}{2f^s}, \quad \forall s. \quad (2b)$$

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<sup>4</sup>Mohring *et Alii* [22] report that, in modelling bus route, it is commonly assumed that, on average, a patron's waiting time for transportation service is half the scheduled headway between subsequent buses. The Authors observe that this assumption might look questionable, if it is considered that regular passengers are likely to know the approximate frequencies at the time they travel. Yet the probability of matching a connection operated by one or the other firm depends on the characteristics of the bus services, rather than on patrons' actions.

<sup>5</sup>In the text, the masculine pronoun (he) is used for the individual customer. At later stage, the feminine pronoun (she) will be introduced for the regulator.

<sup>6</sup>At this stage, it should be clear that, under monopoly, the sole relevant type of passengers is the first one because type-2 behaviour collapses onto type-1. The reader should remark that all passengers have the same preferences, up to their time value and taste parameter, so that (1b) and (1a) are specifications onto which the same surplus function collapses, once individuals are distinguished according to their behaviour.

Both (2a) and (2b) suggest that, at the individual optimum, the utility the consumer derives from the last purchased ticket equals the generalised cost he bears. They as well reveal that, *ceteris paribus*, the taste parameter  $\alpha$  has a positive impact on the individual demand; on the opposite, consumption reduces in the time value  $\tau$ .

Observe that (2a) and (2b) can be used to establish the relationship between demand variations, as induced by changes in firm  $j$ 's price and frequency, assuming that the pair  $(p_k^s, f_k^s)$  remains fixed. Indeed, since (2a) and (2b) hold for any  $p_j^s$ , we can differentiate both of them with respect to  $p_j^s$  and obtain

$$\alpha U'' \frac{\partial x_j^{s,1}}{\partial p_j^s} = 1, \quad \forall s \quad (3a)$$

and

$$\alpha U'' \frac{\partial x_j^{s,2}}{\partial p^{s,e}} = 1, \quad \forall s, \quad (3b)$$

respectively. (3a) and (3b) reveal that a unitary increase in price  $p_j^s$  induces a unitary increase in the marginal utility of the service for the passenger, whatever his behavioural type, through the variation intervened in his demand.

On the other hand, (2a) and (2b) are true for any  $f_j^s$ . Differentiating both of them with respect to this variable returns

$$\alpha U'' \frac{\partial x_j^{s,1}}{\partial f_j^s} = -\frac{\tau}{2(f_j^s)^2}, \quad \forall s \quad (4a)$$

and

$$\alpha U'' \frac{\partial x_j^{s,2}}{\partial f^s} = -\frac{\tau}{2(f^s)^2}, \quad \forall s^7, \quad (4b)$$

respectively. (4a) and (4b) reveals that a unitary increase in frequency induces a reduction equal to  $\tau/2(f_j^s)^2$  in the marginal utility of a type-1 customer, through a change in his demand. Observe that the higher the frequency initially provided by firm  $j$ , the smaller the variation in marginal utility induced by further scheduling. Indeed, when the enterprise already offers very frequent connections, receiving more causes a relatively small increase in the demand for the service. (4b) has a similar interpretation; the sole difference between type-1 and type-2 customers is that for the latter the relevant number of travels is, in fact, the total number of connections available at the industry level.

Let us next combine (3a) with (4a) and (3b) with (4b); this yields the equalities

$$\frac{\partial x_j^{s,1}}{\partial f_j^s} = -\frac{\tau}{2(f_j^s)^2} \frac{\partial x_j^{s,1}}{\partial p_j^s}, \quad \forall s \quad (5a)$$

and

$$\frac{\partial x_j^{s,2}}{\partial f^s} = -\frac{\tau}{2(f^s)^2} \frac{\partial x_j^{s,2}}{\partial p^{s,e}}, \quad \forall s, \quad (5b)$$

which highlight the link between sensitivity of the individual demand to the frequency and to the price.

At this stage on the investigation, it should be evident that the behavioural type of each passenger endogenously follows from his characteristics, which structure the personal preferences. In particular, it is possible to show that a crucial role is played by the

<sup>7</sup>(4b) is found by replacing  $\partial x_j^{s,2}/\partial p^{s,e} = 1/\alpha U''$  from (3b).

individual time value (analytical details are relegated to Appendix A<sup>8</sup>). The following Proposition makes this point explicit.

**Proposition 1** *In a duopolistic shipping industry, there exists a cutoff time value such that people exhibiting smaller  $\tau$  behave as type-1 passengers and patronize the cheaper operator, whereas people with larger  $\tau$  act as type-2 passengers.*

Observe that the relevant cutoff time value, separating type-1 from type-2 passengers, depends on the elements hereafter listed:

1. the wedge between the prices the two operators charge;
2. the frequency offered by the cheaper provider, which is patronized by type-1 customers.

Let us firstly comment on 1. Having a large price gap means that travelling with one firm is much more expensive than it is with the other. This circumstance makes the cheaper operator relatively more convenient for a wider range of time values, hence the marginal value of  $\tau$  moves upward over the total support. Similarly, turning to 2., as the quality supplied by the cheaper operator increases, its service becomes relatively more attractive for a wider interval of time values, which has analogous impact on the position of the cutoff  $\tau$ .

The previous considerations suggest that, for the infra-marginal type-1 customers, the main concern is given by the price paid for travelling. In other words, for those passengers, smaller price is more important, as compared to quality; hence, it is preferred, even when associated with the poorer quality. On the other hand, the amount of connections operated by the cheaper firm matters at the margin, in that it contributes to tilt type-1 behaviour to type-2. Precisely the passage from one behaviour to the other rules out the circumstance that people whose time value is smaller than the cutoff  $\tau$  reduce their demand for transportation service, as they become more likely to use the more expensive connection.

Notice that the individual taste parameter does not directly enter the unit generalised costs that each traveller compares in order to choose at his best. Conversely, the individual time value does have a direct effect, as it shows up in the unit generalised costs. Nevertheless, the prices and frequencies the firms offer (and the single traveller takes as given) actually depend on the joint distribution of  $\alpha$  and  $\tau$  in the population.

One last point we need to make. All along the sequel of our work, the investigation is performed at the aggregate level, because this is the relevant perspective for both firms and regulatory bodies. It is therefore necessary to determine the aggregate demand functions. The analysis so far performed, together with the results achieved in Appendix A, provides us with the appropriate information. In particular, we are able to establish that, whenever it is  $f_j^s > f_k^s$  and  $p_j^s > p_k^s$ , aggregate demand functions are given by

$$X_k^s(\mathbf{p}^s, \mathbf{f}^s) = X_k^{s,1}(p_k^s, f_k^s) + \frac{f_k^s}{f^s} X^{s,2}(\mathbf{p}^s, \mathbf{f}^s), \quad \forall s$$

and

$$X_j^s(\mathbf{p}^s, \mathbf{f}^s) = \frac{f_j^s}{f^s} X^{s,2}(\mathbf{p}^s, \mathbf{f}^s), \quad \forall s$$

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<sup>8</sup>The analytical details reported in Appendix A also allow to derive firms' aggregate demand functions, which we subsequently list in the text.

for firm  $k$  and  $j$  respectively, where one has

$$X_k^{s,1}(p_k^s, f_k^s) = \int_0^{\tau_{mg}^{s,2,k}} \int_{\alpha} x_k^{s,1}(\alpha, \tau; p_k^s, f_k^s) g(\alpha, \tau) d\alpha d\tau, \forall s$$

and

$$X^{s,2}(\mathbf{p}^s, \mathbf{f}^s) = \int_{\tau_{mg}^{s,2,k}}^{+\infty} \int_{\alpha} x^{s,2}(\alpha, \tau; \mathbf{p}^s, \mathbf{f}^s) g(\alpha, \tau) d\alpha d\tau, \forall s.$$

Conversely, with  $f_j^s > f_k^s$  and  $p_j^s < p_k^s$ , demand functions write as

$$X_k^s(\mathbf{p}^s, \mathbf{f}^s) = \frac{f_k^s}{f^s} X^{s,2}(\mathbf{p}^s, \mathbf{f}^s), \forall s$$

and

$$X_j^s(\mathbf{p}^s, \mathbf{f}^s) = X_j^{s,1}(p_j^s, f_j^s) + \frac{f_j^s}{f^s} X^{s,2}(\mathbf{p}^s, \mathbf{f}^s), \forall s,$$

where it is

$$X^{s,2}(\mathbf{p}^s, \mathbf{f}^s) = \int_{\tau_{mg}^{s,2,j}}^{+\infty} \int_{\alpha} x^{s,2}(\alpha, \tau; \mathbf{p}^s, \mathbf{f}^s) g(\alpha, \tau) d\alpha d\tau, \forall s$$

and

$$X_j^{s,1}(p_j^s, f_j^s) = \int_0^{\tau_{mg}^{s,2,j}} \int_{\alpha} x_j^{s,1}(\alpha, \tau; p_j^s, f_j^s) g(\alpha, \tau) d\alpha d\tau, \forall s.$$

One can also compute the aggregate indirect utility functions by plugging the individual demands into the individual surplus functions (1a) and (1b) and then summing up over the relevant ranges of time values in the population. For sake of shortness, we omit this exercise.

## 2.2 The Technologies and Profits

So far we have sketched out the essential characteristics of the demand side of the maritime ferry market. In the present Section, we describe the provision side and, in particular, the most important features of the technologies. Again, for expositional reasons, we look at both operators at once; nevertheless, one should keep in mind that only firm  $I$  matters, in the event that the sector is monopolistic. Moreover, we reintroduce the passenger classification into the two categories (namely, residents and non-residents) we mentioned when we presented the model. We denote by  $X_j^{s,i}(\mathbf{p}^{s,i}, \mathbf{f}^s)$  firm  $j$ 's aggregate demand on market segment  $i$  in season  $s$ ,  $\forall j, s, i$ , which depends on the vector of relevant prices  $\mathbf{p}^{s,i} = (p_j^{s,i}, p_k^{s,i})$  as well as on the vector of relevant frequencies  $\mathbf{f}^s = (f_j^s, f_k^s)$ <sup>9</sup>.

We are now ready to focus on technologies. We assume that, for either operator, the cost function consists in three main components, which we hereafter illustrate.

The first component is purely operational and is to be attributed to the used capacity. More precisely, it includes the costs associated with shipping personnel, passenger transferring, boarding and debarking operations and various related expenses. The utilized capacity, which we denote by  $K_j^s$ , represents the number of seats on firm  $j$ 's ships which

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<sup>9</sup>This notation should not generate a confusion as to aggregate demand functions. The aggregate demand we refer to in the current Section forms precisely as illustrated in the previous Section. The only difference is that we now consider a category-classification, rather than a type-classification.

are occupied in season  $s$ . This capacity depends on both faced traffic  $X_j^s = \sum_i X_j^{s,i}$  and offered connection frequencies  $f_j^s$ ; indeed, it equals the ratio  $X_j^s/f_j^s$ . Observe that, for any given level of traffic, the larger the frequency, the smaller  $K_j^s$ ; in the presence of increasing returns to scale, this involves higher per-passenger cost. The marginal cost of operation is assumed to be constant for either shipper; more precisely, it is given by  $a$  for firm  $E$  and by  $(a + \gamma)$  for firm  $I$  respectively. The hypothesis that the incumbent has larger marginal cost is in line with Cremer *et Alii* [9]; the latter capture the fact that equally skilled workers are frequently over-remunerated in public enterprises through the hypothesis that the latter pay a premium to their employees, an extra cost which appears as a budget component<sup>10</sup>. The total per-year costs associated with the used capacity amount to  $aK_E^s f_E^s = a \sum_s X_E^s$  for the entrant and to  $(a + \gamma) \sum_s K_I^s f_I^s = (a + \gamma) \sum_s X_I^s$  for the dominant operator respectively. Hence, this cost component proportionally increases in the traffic size.

The second component is specifically associated with the number of travels performed with the available capacity, independently of whether the latter is fully occupied or remains (partially) idle. For instance, the activities related to mooring and sailing are executed at each travel, no matter how many passengers occupy the seats. In the long run, shippers adjust installed capacity according to the observed traffic, taking into account that, in the short run, they will benefit from seasonal flexibility in frequency. Therefore, installed capacity is finally equivalent to  $\text{Sup} \{K_j^l, K_j^h\} \equiv \bar{K}_j$ , that is to the capacity that is actually used in the season during which no excess is registered<sup>11</sup>. We assume that it generates a cost  $\phi_j(\bar{K}_j)$  so that the overall associated burden amounts to  $\phi_j(\bar{K}_j) \sum_s f_j^s$ . We also suppose that it is  $\phi_E > \phi_I$ . Hence, while the incumbent is operationally less efficient than the entrant, it exhibits a cost advantage in terms of capital. This is explained if one recalls that, in the real-world sectors we refer to, the dominant enterprise is frequently the statutory provider, formerly or still public. Such a status is perceived to be a guarantee for repayment, hence it helps obtain better financing conditions, which translates into lower cost of capital. This is relevant because, beyond some amount of frequencies, providing further connections requires having larger fleets. Under our assumption, disposing of bigger capacity is relatively more affordable for shipper  $I$ <sup>12</sup>.

Thirdly, each firm bears a pure fixed cost  $F_j$ , mainly associated with maintenance of ships and accessory equipment as well as to administration, advertising, insurance. Hence, it is to be incurred even when no travel is performed.

Finally, letting  $\sum_{s,i} X_j^{s,i} p_j^{s,i}$ ,  $\forall j$ , represent the total revenues firm  $j$ 's service generates all over the year on the two market segments and putting things together, we can write shipper  $I$ 's yearly profit function as

$$\pi_I(\mathbf{p}, \mathbf{f}) = \sum_{s,i} X_I^{s,i} p_I^{s,i} - \left[ (a + \gamma) \sum_s X_I^s + \phi_I \sum_s f_I^s + F_I \right], \quad (8a)$$

<sup>10</sup> Martimort [21] reports that, according to Lopez-de-Silvanes *et Alii* (1997), wages in the public sector are 10 to 20 percent higher than those that are paid for similar jobs in the private sector. This matter of fact partially explains the wave of strikes that perturbed the French ferry service during fall 2005, when the employees of the public shipping company SNCM strongly opposed the French government's intention to privatize the firm.

<sup>11</sup> At the operational stage, the firm's cost function is, in fact, a short-run function. The size of capacity is a matter of long-run strategy and should be viewed as the first decision variable in a two-stage game in which enterprises anticipate the subsequent price-and-frequency choice.

<sup>12</sup> Martimort [21] points that firms which lack reputational capital, as the entrant in our shipping industry, may experience some difficulties at accessing financial markets.

whereas operator  $E'$ 's is given by

$$\pi_E(\mathbf{p}, \mathbf{f}) = \sum_{s,i} X_E^{s,i} p_E^{s,i} - \left( a \sum_s X_E^s + \phi_E \sum_s f_E^s + F_E \right). \quad (8b)$$

Each of the previous functions is twice continuously differentiable and strictly concave everywhere in the firm's actions.

### 3 The Utilitarian Social Optimum

In the previous Section, we have outlined the relevant demand and cost features of the maritime ferry market. In what follows, we explore the first-best benchmark for the sector under scrutiny. We initially analyze the monopoly case. At later stage, we focus on the duopoly environment.

#### 3.1 The First Best under Monopoly

Let us begin by considering the case where the shipping industry is served by a monopolistic firm. Within this scenario, we hereafter characterize the prices and frequencies which maximize the social welfare function

$$W^M(\mathbf{p}_I, \mathbf{f}_I; \boldsymbol{\alpha}, \boldsymbol{\tau}) = V(\mathbf{p}_I, \mathbf{f}_I; \boldsymbol{\alpha}, \boldsymbol{\tau}) + \pi_I(\mathbf{p}_I, \mathbf{f}_I), \quad (9)$$

which is taken to be the *unweighed* sum of aggregate consumer surplus  $V(\cdot) = \sum_{s,i} V^{s,i}$ <sup>13</sup> and monopoly profits  $\pi_I(\mathbf{p}_I, \mathbf{f}_I)$ , where  $\mathbf{p}_I = (p_I^{s,i})_{s,i}$ ,  $\mathbf{f}_I = (f_I^s)_{s,i}$  and  $M$  stays for *monopoly*.

The utilitarian functional form in (9) captures the circumstance that, for the time being, efficiency is taken to be the sole relevant scope. Moreover, at this stage, the provider is not required to break even. One may imagine that its participation in the market operation be ensured under the hypothesis that the government covers the extra costs (including the cost of capital) from the general budget, by providing subsidies at no cost of public funds.

The first-order condition of  $W^M$  with respect to  $p_I^{s,i}$  writes as

$$p_I^{M,FB} = a + \gamma, \quad (10)$$

where the superscript *FB* means *first best*. (10) shows that price should equal marginal cost. Observe that, the first-best tariff remains constant all over the year, as the marginal cost stays the same whatever the season. Moreover the price does not reflect the heterogeneity characterizing the population, hence it is equal for both categories of travellers.

Furthermore, using the marginal cost pricing rule in (10), the first-order condition of  $W^M$  with respect to  $f_I^s$  is given by

$$\frac{\partial V^s}{\partial f_I^s} = \phi_I, \quad \forall s,$$

which is equivalent to

$$\int \int \frac{\tau}{\alpha} \frac{1}{2(f_I^s)^2} \sum_i x_I^{s,i} g(\alpha, \tau) d\alpha d\tau = \phi_I, \quad \forall s. \quad (11)$$

<sup>13</sup>  $V^{s,i}$  is the aggregate indirect utility function of category  $i$  in season  $s$ , as mentioned (but omitted) at the end of Section 2.1.  $V(\cdot)$  sums up over categories and seasons.

(11) states the equality between marginal benefit and marginal cost of connection, suggesting that, at the social optimum, shipper  $I$  should increase frequency until the additional benefit to consumers, which is generated by the last provided connection, is fully offset by the incremental cost it imposes on the operator. Observe that, differently from the price, the first-best frequency may well adjust on a seasonal basis, as it is determined not only by the firm's technology, but also by the demand characteristics of the market.

To save over notation, we set  $\tilde{\tau}^{s,M,FB} \equiv \int \int \tau \sum_i x_I^{s,i} g(\alpha, \tau) d\alpha d\tau$ . We then rewrite (11) as

$$\frac{\tilde{\tau}^{s,M,FB}}{2(f_I^s)^2} = \phi_I, \quad \forall s,$$

which finally yields

$$f_I^{s,M,FB} = \sqrt{\frac{\tilde{\tau}^{s,M,FB}}{2\phi_I}}, \quad \forall s. \quad (12)$$

(12) provides the socially optimal number of connections under monopoly as a relatively simple expression. In particular, firm  $I$ 's first-best frequency equals the square root of a ratio which has a weighted sum of the individual demands on the two market segments at the numerator, weights being the time values, and twice the marginal cost of a connection at the denominator.

### 3.2 The First Best under Duopoly

We now move to the duopolistic environment, where the social welfare function becomes

$$W^D(\mathbf{p}, \mathbf{f}; \boldsymbol{\alpha}, \boldsymbol{\tau}) = V(\mathbf{p}, \mathbf{f}; \boldsymbol{\alpha}, \boldsymbol{\tau}) + \sum_{j=I, E} \pi_j(\mathbf{p}, \mathbf{f}). \quad (13)$$

The one in (13) is still a utilitarian function. Nevertheless, as compared to (9), the welfare of the collectivity, namely  $W^D$ , the superscript  $D$  staying for *duopoly*, additionally contains firm  $E$ 's profits ( $\pi_E$ ).

Before determining the first-best prices and frequencies under duopoly, we find it important to establish when and whether it is socially optimal that either firm operates, given the cost structures. For this purpose, we need to compare shippers' per-passenger costs, as obtained by dividing variable costs by total traffics. More precisely, we have

$$PPVC_I^s = a + \gamma + \frac{\phi_I f_I^s}{X_I^s} \quad (14a)$$

for the incumbent and

$$PPVC_E^s = a + \frac{\phi_E f_E^s}{X_E^s} \quad (14b)$$

for the entrant<sup>14</sup>. For the industry per-passenger variable cost to be minimized, firm  $I$

<sup>14</sup>In the text, we abstain from considering the fixed cost components for two reasons. Firstly, at least in a short-run perspective, fixed costs are sunk and do not affect the optimal allocation. Secondly, in a first-best environment, shippers are not required to be viable in the long run without public financing. Clearly, in a second-best world with budget balance requirements, things would differ. If the social planner can decide whether to have one or two operators in the shipping market, then the presence of fixed costs does affect the ultimate choice, to the extent that, once the decision is made, all active firms need to break-even without relying on public resources. See Cremer *et Alii* [9] for a similar argument. See also Laffont [19] for a more general discussion as to how duplication of fixed costs may lead to sub-optimal allocations.

should operate for all the values of  $X_I^s$ ,  $X_E^s$ ,  $f_I^s$  and  $f_E^s$  such that, given  $\gamma$ ,  $\phi_I$  and  $\phi_E$ , it is

$$PPVC_I^s < PPVC_E^s \Leftrightarrow \gamma < \left( \frac{\phi_E f_E^s}{X_E^s} - \frac{\phi_I f_I^s}{X_I^s} \right). \quad (15)$$

Provided that  $\gamma > 0$ , a necessary condition for (15) to hold is given by  $\phi_E f_E^s / X_E^s > \phi_I f_I^s / X_I^s$ , meaning that the entrant's per-passenger cost of connection in season  $s$  must exceed the dominant enterprise's. Observing that  $\gamma$  measures the difference between shippers' per-passenger operational costs, one concludes that (15) is satisfied whenever the additional per-passenger cost firm  $I$  imposes on society in terms of operation, as compared to firm  $E$ , is smaller than the per-passenger cost savings it allows for in terms of connections. Under this circumstance, service provision by the dominant operator yields a net per-passenger benefit, hence it is relatively more desirable for the collectivity<sup>15</sup>.

Clearly, the condition  $\phi_E f_E^s / X_E^s > \phi_I f_I^s / X_I^s$  is not sufficient for shipper  $I$  to dominate in a first-best environment; according to (15), enterprise  $E$  rather dominates for  $\gamma$  sufficiently large. In particular, given capacities, the value of  $\gamma$  triggering the entrant's preferability depends on the discrepancy between  $\phi_E$  and  $\phi_I$ .

Furthermore, it is better to solely entitle firm  $E$  with the provision of transportation service whenever one has  $\phi_E f_E^s / X_E^s < \phi_I f_I^s / X_I^s$ , in which case (15) cannot be met. In this scenario, firm  $I$  exhibits both higher per-passenger cost of frequency and higher per-passenger cost of operation. Therefore, letting this shipper supply the service would generate a net per-passenger penalty, which is not induced, instead, by the other provider.

It is now time to determine the optimal pricing and scheduling for the duopoly. Marginal cost pricing entails for either operator, so that we have

$$p_I^{D,FB} = a + \gamma \quad (16a)$$

and

$$p_E^{D,FB} = a \quad (16b)$$

for firm  $I$  and  $E$  respectively. According to (16a) and (16b), efficiency requires that prices only reflect technologies. The reader should recall that this happens also under monopoly. Hence, the first-best price for firm  $I$  is the same, whatever the market structure. From (16a) and (16b), we clearly have  $p_I^{D,FB} > p_E^{D,FB}$ , the difference between tariffs expressing the gap in marginal costs ( $\gamma$ ). At the first best, the duopoly average price is unambiguously lower than the monopoly one.

Applying the marginal cost pricing rules, we find that, as under monopoly, the optimal scheduling is characterized by the equality between marginal benefit and marginal cost of connection for either shipper, namely by

$$\frac{\partial V^s}{\partial f_j^s} = \phi_j, \quad \forall s, j.$$

In order to have a specification of the above condition for the duopoly scenario, we need to rely on the results summarized in our first Proposition. Indeed, the latter allows to deduce how travellers allocate between operators in a first-best environment with both firms active. In particular, the relevant cutoff time value is equal to  $2\gamma f_E^{s,FB}$ ; passengers whose  $\tau \in [0, 2\gamma f_E^{s,FB}]$  patronize the entrant, those with  $\tau \in (2\gamma f_E^{s,FB}, +\infty)$  take the ship sailing next<sup>16</sup>. As one may recall, this is so because, when the time value is little, the most relevant element resides in the price. Since shipper  $E$  offers the cheaper service,

<sup>15</sup>A special case arises when shippers have the same level of used capacities, so that the right-hand side of (15) is necessarily positive.

<sup>16</sup>In the first-best environment, type-2 passengers perceive the price to be equal to  $p^{s,e,FB} = a + \gamma f_I^s / f^s$ .

this is the operator type-1 customers prefer. Saving over time becomes more important as the penalty from waiting gets larger; then passengers are better off by departing as soon as possible, which leaves room to both shippers' activities. In this perspective, operation by the dominant enterprise appears essentially beneficial to type-2 customers, to whom it provides additional connections.

From the above considerations it follows that the optimal scheduling rule for firm  $I$  specifies as

$$\int_{2\gamma f_E^s}^{+\infty} \int_{\alpha} \left[ \frac{\tau}{2(f^s)^2} - \frac{\gamma f_E^s}{(f^s)^2} \right] \sum_i x^{s,2,i} g(\alpha, \tau) d\alpha d\tau = \phi_I, \quad \forall s, \quad (17)$$

where the ratio  $\gamma f_E^s / (f^s)^2$  is the derivative of the perceived price  $p^{s,e,FB}$  with respect to frequency  $f_I^s$ . It is positive, i.e. the perceived price increases in shipper  $I$ 's number of connections because, at the first best, firm  $I$ 's service is more expensive than firm  $E$ 's.

The interpretation of the left-hand side of (17) is rather interesting. The ratio  $\tau/2(f^s)^2$  measures the *gross* variation in type-2 passenger utility, which is caused by the increase in the total number of available connections. A second effect of further scheduling shows up through the ratio  $\gamma f_E^s / (f^s)^2$ . To help the economic comprehension of the latter, it is useful to observe that it is the product of two terms, namely  $\gamma/f^s$  and  $f_E^s/f^s$ . To begin with, we notice that we have  $(p_I^{D,FB} - p_E^{D,FB})/f^s = \gamma/f^s$ , suggesting that this term represents the per-travel price wedge. Furthermore, the ratio  $f_E^s/f^s$  is the portion of connections provided by shipper  $E$  in the industry. Therefore, the product of the two terms synthesizes the penalty to be borne by the concerned passengers when it becomes more likely to forgo the cheaper connections as the frequency of the more expensive firm (namely,  $f_I^s$ ) increases. In definitive, the difference  $\tau/2(f^s)^2 - \gamma f_E^s / (f^s)^2$ , that is the gross variation diminished by the penalty, expresses the *net* individual marginal utility of firm  $I$ 's scheduling per unit of consumption. It is now straightforward to see that the left-hand side of (17) aggregates net marginal utilities for all travels made by all concerned passengers.

Remarkably, (17) can be used to determine the *total* number of connections in the industry, not to explicitly deduce shipper  $I$ 's frequency. This hinges on the circumstance that firm  $I$  serves type-2 passengers only. To establish the optimal frequency for the sector, we first rewrite (17) as

$$\frac{\tilde{\tau}^{s,D,FB}}{2(f^s)^2} = \phi_I, \quad \forall s,$$

where we have introduced the definition  $\tilde{\tau}^{s,D,FB} \equiv \int_{2\gamma f_E^s}^{+\infty} \int_{\alpha} (\tau - 2\gamma f_E^s) \sum_i x^{s,2,i} g(\alpha, \tau) d\alpha d\tau$ .

It is then straightforward to obtain

$$f^{s,D,FB} = \sqrt{\frac{\tilde{\tau}^{s,D,FB}}{2\phi_I}}, \quad \forall s, \quad (18)$$

Observe that the numerator of the ratio under square root in (18) is a weighted sum of the demands expressed by the type-2 passengers belonging to both categories, weights being the deviations of the individual time values from the cutoff value.

As to firm  $E$ 's scheduling, the first-order condition writes as

$$\begin{aligned}\phi_E &= \int_0^{2\gamma f_E^s} \int_{\alpha} \frac{\tau}{2(f_E^s)^2} \sum_i x_E^{s,1,i} g(\alpha, \tau) d\alpha d\tau \\ &+ \int_{2\gamma f_E^s}^{+\infty} \int_{\alpha} \left[ \frac{\tau}{2(f^s)^2} + \frac{\gamma f_I^s}{(f^s)^2} \right] \sum_i x_E^{s,2,i} g(\alpha, \tau) d\alpha d\tau, \forall s.\end{aligned}\quad (19)$$

Two terms are summed up in the right-hand side of (19) because shipper  $E$  serves both types of travellers<sup>17</sup>. We hereafter illustrate each of them in details.

The first term, namely  $\int_0^{2\gamma f_E^s} \int_{\alpha} \left[ \tau/2(f_E^s)^2 \right] \sum_i x_E^{s,1,i} g(\alpha, \tau) d\alpha d\tau$ , expresses the variation induced in the utility of type-1 passengers as  $f_E^s$  increases.

Similarly, the second term, i.e.  $\int_{2\gamma f_E^s}^{+\infty} \int_{\alpha} \left[ \tau/2(f^s)^2 + \gamma f_I^s / (f^s)^2 \right] \sum_i x_E^{s,2,i} g(\alpha, \tau) d\alpha d\tau$ ,

represents the variation induced in the utility of type-2 passengers as  $f_E^s$  gets larger. Yet this term deserves a few more words. The ratio  $\tau/2(f^s)^2$  is to be interpreted as we did for (17). On the other hand, the ratio  $\gamma f_I^s / (f^s)^2$  is the (absolute value of the) derivative of the first-best perceived price with respect to  $f_E^s$ <sup>18</sup>. The term under scrutiny differs from the one in (17) in that the per-travel price wedge  $(p_I^{D,FB} - p_E^{D,FB}) / f^s = \gamma / f^s$  is now multiplied by the portion of connections provided by shipper  $I$ , namely  $f_I^s / f^s$ , and the positive sign appears in front of it. The product of these terms expresses the savings that become available to the concerned passengers, when it becomes more likely to take the cheaper connections as the frequency of the less expensive firm (namely,  $f_E^s$ ) increases. Therefore, the sum  $\tau/2(f^s)^2 + \gamma f_I^s / (f^s)^2$ , i.e. the basic marginal utility of  $f_E^s$  augmented by the bonus for service  $E$  being cheaper, expresses the *overall* individual marginal utility of firm  $E$ 's scheduling per unit of consumption. The latter is then aggregated for all travels made by all concerned passengers.

We next rewrite (19) as

$$\phi_E = \frac{\tilde{\tau}^{s,D,FB}}{2(f_E^s)^2} + \frac{\tilde{\tau}^{s,D,FB}}{2(f^s)^2}, \forall s,$$

where we have defined  $\tilde{\tau}^{s,D,FB} \equiv \int_0^{2\gamma f_E^s} \int_{\alpha} \tau \sum_i x_E^{s,1,i} g(\alpha, \tau) d\alpha d\tau$  together with  $\tilde{\tau}^{s,D,FB} \equiv \int_{2\gamma f_E^s}^{+\infty} \int_{\alpha} (\tau + 2\gamma f_I^s) \sum_i x_E^{s,2,i} g(\alpha, \tau) d\alpha d\tau$ . Then the amount of connections shipper  $E$  should provide at the first best can be determined as

$$f_E^{s,D,FB} = \sqrt{\frac{\tilde{\tau}^{s,D,FB}}{2\phi_E - \frac{\tilde{\tau}^{s,D,FB}}{f^{s,D,FB}}}} = \sqrt{\frac{\tilde{\tau}^{s,D,FB}}{2\phi_E - \frac{\tilde{\tau}^{s,D,FB}}{\sqrt{\frac{\tilde{\tau}^{s,D,FB}}{2\phi_I}}}}}, \forall s. \quad (20)$$

<sup>17</sup>The terms showing up in the condition in the text are those which express the impact of a change in  $f_E^s$  on the infra-marginal units of traffic. On the opposite, the marginal variations do not appear as they cancel out. Indeed, one has  $\int_{\alpha} \sum_i [\alpha U(x_E^{s,1,i} (2\gamma f_E^s)) - (a + \gamma) x_E^{s,1,i} (2\gamma f_E^s)] g_2(\alpha, 2\gamma f_E^s) d\alpha = \int_{\alpha} \sum_i [\alpha U(x_E^{s,2,i} (2\gamma f_E^s)) - (a + \gamma) x_E^{s,2,i} (2\gamma f_E^s)] g_2(\alpha, 2\gamma f_E^s) d\alpha$ , where  $g_2(\alpha, 2\gamma f_E^s)$  is the marginal density of  $\tau$ , evaluated at  $2\gamma f_E^s$ .

<sup>18</sup>To be precise, one has  $\partial p^{s,e,FB} / \partial f_E^s = -\gamma f_I^s / (f^s)^2$ . This value is negative, meaning that the price type-2 passengers perceive at first best decreases in firm  $E$ 's number of connections, because shipper  $E$  provides the cheaper service.

Our final step consists in subtracting (20) from (18), at the aim of establishing the socially optimal number of travals for firm  $I$ . The latter is given by

$$f_I^{s,D,FB} = f^{s,D,FB} - f_E^{s,D,FB} = \sqrt{\frac{\tilde{\tau}^{s,D,FB}}{2\phi_I}} - \sqrt{\frac{\tilde{\tau}^{s,D,FB}}{2\phi_E - \frac{\tilde{\tau}^{s,D,FB}}{\sqrt{\frac{\tilde{\tau}^{s,D,FB}}{2\phi_I}}}}}, \forall s. \quad (21)$$

The analysis so far performed shows that the expression for socially optimal pricing is as simple under duopoly as it is when passengers are served by a monopolist. On the other hand, the expression for first-best scheduling is more complex when two shippers operate than it is in the presence of a unique provider. Marginal cost pricing entails, hence pricing solely reflects technological conditions, whatever the sector structure. It is then clear that the complication comes from the demand side of the market. In particular, it hinges on the circumstance that duopolistic service provision allows the more impatient travellers to randomize over the two shippers for reducing their waiting time whereas, under monopoly, everybody is forced to programme his travals (that is, to behave as a type-1 customer). Under duopoly, both a dimension of horizontal and one of vertical differentiation show up, which significantly enrich the environment and complicate the investigation.

## 4 The Regulated Monopoly

In the previous Section, we pointed that, for conditions (10) and (11) to become attainable, it should be possible to fund the uncovered costs of provision by means of subventions taken from the general budget of the State without creating efficiency losses. In reality, this does not seem to be feasible because, in general, resource collection requires levying distorting taxes. Therefore, the rules listed above remain ideal reference points.

It is now time to concentrate on more realistic scenarios. In the present Section, we focus on a monopolistic ferry industry in which the shipper is compelled to implement the policy the regulator designs. This situation has persistently had, and still often has, undeniable practical relevance in most European countries.

In the framework under scrutiny, the unique shipper (firm  $I$ ) is instructed to pursue the social interests compatibly with budget balance. As long as society aims at achieving efficiency, solving the social problem amounts to maximizing the utilitarian welfare function under the constraint that overall profits be non-negative. The programme writes as

$$\begin{aligned} & \underset{\{p_I^{s,i}, f_I^s\}_{\forall s,i}}{\text{Max}} \quad V(\mathbf{p}_I, \mathbf{f}_I; \boldsymbol{\alpha}, \boldsymbol{\tau}) + \pi_I(\mathbf{p}_I, \mathbf{f}_I) \\ & \text{subject to} \\ & \pi_I(\mathbf{p}_I, \mathbf{f}_I) \geq 0, \end{aligned} \quad (22)$$

where  $\mathbf{p}_I = (p_I^{h,r}, p_I^{h,n}, p_I^{l,r}, p_I^{l,n})$  and  $\mathbf{f}_I = (f_I^h, f_I^l)$  are the vectors of prices and frequencies to be regulated. Observe that we do not require that each market segment and season be self-financing, which would rather call for separated budget constraints. This would be problematic as it would require the adoption of appropriate rules for the allocation of the common cost components.

Let  $\lambda^{RM}$  the Lagrange multiplier which quantifies the effect that is induced by a variation in the fixed cost included in the budget constraint on the optimal value of the objective function. The superscript  $RM$  is meant to indicate the *regulated monopoly* regime. The first-order conditions which characterize the (constrained) optimal prices

$p_I^{s,i,RM}$  and frequencies  $f_I^{s,RM}$  are given by

$$\frac{\partial \pi_I}{\partial p_I^{s,i}} = -\frac{\partial V^{s,i}}{\partial p_I^{s,i}} \frac{1}{1 + \lambda^{RM}}, \quad \forall s, i, \quad (23a)$$

and

$$-\frac{\partial \pi_I}{\partial f_I^s} = \frac{\partial V^s}{\partial f_I^s} \frac{1}{1 + \lambda^{RM}}, \quad \forall s, \quad (23b)$$

respectively. (23a) means that the incremental profits firm  $I$  obtains on the last unit increase in price should equal the reduction induced in consumer surplus, discounted according to the shadow value of the budget constraint. Similarly, (23b) suggests that the decrease in the shipper's profits over the last performed travel oughts to equal the associated increment in consumer surplus, again discounted by the cost  $\lambda^{RM}$ .

Altogether, (23a) and (23b) synthesize how, in the words of the Regulation 3577/92 [17], the authority forces the firm to "obligations which, if considering its own commercial interest, it would not assume". Combining the two conditions yields

$$\frac{\partial \pi_I / \partial f_I^s}{\partial \pi_I / \partial p_I^{s,i}} = \frac{\partial V^s / \partial f_I^s}{\partial V^s / \partial p_I^{s,i}}, \quad \forall s, i. \quad (24)$$

The left-hand side of (24) is the rate at which price and frequency can be substituted away for the shipper benefits to remain unchanged<sup>19</sup>. Similarly, the right-hand side is the rate of substitution between frequency and price, such that consumer surplus is left unaffected<sup>20</sup>. Overall, (24) suggests that, by equalizing these rates, the least possible amount of social welfare is sacrificed to the monopolist's budgetary requirements.

Furthermore, defining  $\varepsilon_I^{(s,i)(s,i)} \equiv (p_I^{s,i} / X_I^{s,i}) (-\partial X_I^{s,i} / \partial p_I^{s,i})$  the (absolute value of the) elasticity of demand  $X_I^{s,i}$  to own price  $p_I^{s,i}$ , (23a) becomes

$$\frac{p_I^{s,i} - (a + \gamma)}{p_I^{s,i}} = \frac{1}{\varepsilon_I^{(s,i)(s,i)}} \frac{\lambda^{RM}}{1 + \lambda^{RM}}, \quad \forall s, i. \quad (25)$$

(25) identifies the Ramsey-Boiteux criterion. The latter ensures that production costs are just covered and that the welfare loss associated with consumption rationing is minimized, so that a socially desirable compromise entails between social welfare and shipper's viability.

Observe that the price  $p_I^{s,i,RM}$  reflects now both market and technological conditions. Indeed, the relative margin associated with segment  $i$  and season  $s$  is directly proportional to the term  $\lambda^{RM} / (1 + \lambda^{RM})$ , which depends on costs. Moreover, it is inversely proportional to the price elasticity of demand  $X_I^{s,i}$ ; this means that the more the demand is price elastic, the more the adverse impact of a price increase becomes important. Clearly, as prices are related to demand elasticities, they depend on the distribution of passenger individual characteristics. This would not be the case in a first-best environment where, as highlighted in the previous Section, they would solely reflect the technological conditions. Interestingly, even if the aim of the regulator is to achieve the highest feasible social welfare, and not to maximize the firm's profits, the demand is embodied in the pricing rule only when the shipper's budget constraint is imposed.

Prices depend on quality as well. Nevertheless, the *norm* governing their optimal choice does not vary when frequency is simultaneously selected. Rewriting (23b) more

<sup>19</sup>As long as the budget constraint is binding, this means that the shipper benefits remain equal to zero.

<sup>20</sup>In (24), we use the derivative  $\partial V^s / \partial p_I^{s,i}$  so that the right-hand side of the equality is the rate of substitution we illustrate in the text. Notice, however, that the derivative is equal to  $\partial V^{s,i} / \partial p_I^{s,i}$ , as aggregate consumer surplus is additive in  $s$  and  $i$ .

extensively as

$$-\left\{\sum_i \left[p_I^{s,i} - (a + \gamma)\right] \frac{\partial X_I^{s,i}}{\partial f_I^s} - \phi_I\right\} = \frac{\partial V^s}{\partial f_I^s} \frac{1}{1 + \lambda^{RM}}, \quad \forall s,$$

makes it explicit how the prices charged on the two market segments, hence the respective margins  $\left[p_I^{s,i} - (a + \gamma)\right]$ , are tied to finance the common cost of quality  $\phi_I$ , taking into account the marginal impact of quality on discounted consumer surplus. Moreover, as we did in the first-best scenario, we can use the previous condition to determine the optimal number of connections for the regulated monopolist. The latter is given by

$$f_I^{s,RM} = \sqrt{\frac{\tilde{\tau}^{s,RM} + \lambda^{RM} \sum_i X_I^{s,i} \tilde{\tau}^{s,i,RM}}{2\phi_I (1 + \lambda^{RM})}}, \quad \forall s, \quad (26)$$

where we have used the definitions  $\tilde{\tau}^{s,RM} \equiv \int \int \tau \sum_i x_I^{s,i} g(\alpha, \tau) d\alpha d\tau$  and  $\tilde{\tau}^{s,i,RM} \equiv \int \int \tau \frac{\partial x_I^{s,i} / \partial p_I^{s,i}}{\partial X_I^{s,i} / \partial p_I^{s,i}} g(\alpha, \tau) d\alpha d\tau$  (see Appendix B for details). (26) reveals that the efficient amount of travels equals the square root of a ratio which has a "weighed sum" of the disutilities of waiting time at the numerator and twice the cost of one such travel at the denominator. Furthermore, the ratio accounts for the shadow cost of the budget constraint. The latter enters the weight of one component of the weighed sum at the numerator, but also contributes to deflate the overall ratio.

It is finally noteworthy that the (constrained) optimal amount of connections may well differ across seasons. One possibility is that relatively lower frequency be ensured in the low season, when traffic appreciably shrinks albeit, in the regulated environment, connections are no longer as rare as they would in an unregulated industry.

#### 4.1 Decentralization through a Global Price-and-Frequency Constraint

Conditions (23a) and (23b) characterize the prices and the number of connections that are chosen by a utilitarian welfare-maximizing informed regulator, as long as the shipping industry has monopoly structure. These (constrained) optimal prices and frequencies can be decentralized to a profit-maximizing operator by imposing the quality-adjusted price cap proposed by De Fraja and Iozzi [10]. In what follows, we briefly illustrate how this mechanism applies to the specific context of the maritime ferry sector.

The regulator requires firm  $I$  to satisfy a constraint, which sets an upper bound on the difference between a weighed average of the charged prices and a weighed average of the amount of operated travels. Both the bound and the weights are exogenously determined. In formal terms, the operator's programme writes as

$$\begin{aligned} & \underset{\{p_I^{s,i}, f_I^s\}_{\forall s,i}}{\text{Max}} \quad \pi_I(\mathbf{p}_I, \mathbf{f}_I) \\ & \text{subject to} \\ & \sum_{s,i} \beta_{DMR}^{s,i} p_I^{s,i} - \sum_s \alpha_{DMR}^s f_I^s \leq P_{DMR}, \end{aligned} \quad (27)$$

where  $\beta_{DMR}^{s,i}$  and  $\alpha_{DMR}^s$  are the weights attributed to prices and frequencies respectively and  $P_{DMR}$  is the upper bound. The script *DMR* stays for *decentralized monopoly regulation*.

As De Fraja and Iozzi [10] explain, by attributing a positive weight to frequency  $f_I^s$  ( $\alpha_{DMR}^s > 0$ ), the shipper is induced to increase this quality dimension. Indeed, by doing

so, a change is triggered in the price constraint, which allows for an increment in the average price  $\sum_{s,i} \beta_{DMR}^{s,i} p_I^{s,i}$ . Conversely, omitting the average frequency component would provide an incentive to the firm to shirk on quality for the purpose of reducing costs, so that larger stake would residue under the price cap<sup>21</sup>.

The first-order conditions of (27) with respect to prices and frequencies are respectively given by

$$\frac{\partial \pi_I}{\partial p_I^{s,i}} = \lambda^{DMR} \beta_{DMR}^{s,i}, \quad \forall s, i \quad (28a)$$

and

$$-\frac{\partial \pi_I}{\partial f_I^s} = \lambda^{DMR} \alpha_{DMR}^s, \quad \forall s, \quad (28b)$$

$\lambda^{DMR}$  being the Lagrange multiplier associated with the regulatory constraint. For the choice of the vector  $(\mathbf{p}_I^{RM}, \mathbf{f}_I^{RM})$  to be decentralized, such vector has to solve (28a) and (28b) for the appropriate value of  $\lambda^{DMR}$ . This is the case whenever the equality

$$\lambda^{DMR} = \frac{1}{1 + \lambda^{RM}} \quad (29a)$$

holds together with

$$\beta_{DMR}^{s,i} = -\frac{\partial V^{s,i,RM}}{\partial p_I^{s,i}} = X_I^{s,i,RM}, \quad \forall s, i \quad (29b)$$

and

$$\alpha_{DMR}^s = \frac{\partial V^{s,RM}}{\partial f_I^s} = \frac{\tilde{\tau}^{s,RM}}{2(f_I^{s,RM})^2}, \quad \forall s. \quad (29c)$$

(29b) reveals that the appropriate weight for each price consists in the value the aggregate marginal surplus attains at the regulated solution  $(-\partial V^{s,i,RM} / \partial p_I^{s,i})$ ; in our quasi-linear world, such value coincides with the level of the aggregate demand  $X_I^{s,i,RM}$ . This constitutes the standard result which is found when global price caps are designed. For instance, Billette de Villemeur *et Alii* [4] prove that it holds for a postal sector in which mail distribution is performed together with a composite activity. Nevertheless, their constraint is a pure price cap as, in their framework, no quality dimension is considered. Conversely, the latter represents a crucial peculiarity of the maritime ferry industry.

As (29c) suggests, the appropriate weight for each quality dimension is given by the marginal net benefit consumers obtain at the regulated monopoly solution  $(\partial V^{s,RM} / \partial f_I^s)$ . The latter equals the marginal "savings" in aggregate disutility from waiting, as expressed by the ratio  $\tilde{\tau}^{s,RM} / 2(f_I^{s,RM})^2$ .

Once weights are set as in (29b) and (29c), for (29a) to be met, it suffices to choose the value of the upper bound  $P_{DMR}$  which saturates the regulatory constraint<sup>22</sup>.

<sup>21</sup> Billette de Villemeur [3] as well proposes a price-and-frequency cap for the purpose of implementing the second-best allocation in a monopoly providing air transportation. He formulates the constraint so that the generalised price paid by consumers (that is, the sum of monetary price and disutility from waiting) is smaller than an exogenously set upper bound. De Fraja and Iozzi [10]'s more general approach better suits the present context, as multiple prices and frequencies are here to be delegated.

<sup>22</sup> De Fraja and Iozzi [10] further show how their quality-adjusted price cap translates into two constraints (namely, the quality adjusted Vogelsang-Finsinger constraint and the distance constraint), which allow for a practical (low informationally demanding) implementation of the theoretical cap.

## 5 The Partially Regulated Duopoly

Under the EU Regulation 3577/92 [17], the principle of service freedom has been extended to maritime transportation as from 1999. Regular passenger transport services, ferry transport and cabotage services with the islands of five Member States of the European Union (Spain, France, Italy, Portugal and Greece<sup>23</sup>) have been opened up to all the beneficiaries of the Regulation, namely "the Community shipowners who have their ships registered in, and flying the flag of a Member State, provided that these ships comply with all conditions for carrying out cabotage in that Member State" (Art. 1).

Yet the persisting opportunity of regulating the ferry sector is recognized "in cases where the operation of market forces would not ensure a sufficient service level" (Art. 9 of the Guidelines on State Aid to the Maritime Sector [16]). Under such circumstances, the imposition of regulatory obligations for the provision of scheduled services is considered to be compatible with liberalized environments.

As for a plurality of other utilities opened up to competition, entry of new operators in the shipping industry is expected to follow and, indeed, it has sometimes followed from liberalization and deregulation, thereby leading to *partially regulated oligopolies*. Nevertheless, this phenomenon does not occur systematically. Anecdotal evidence suggests that the scope (eventually) left for profitable access to markets where regulated incumbents rely on solid customer bases sensibly differs, depending on the adopted policies.

As long as a regulated industry is concerned, it is of crucial importance to understand how this circumstance hinges on the specific institutional features. Indeed, a necessary condition for the regulatory policy to be properly designed is that its impact on the surrounding and perspective environment be taken into account. Some of the Sections which follow are actually meant to assess how regulation of a dominant firm (shipper  $I$ ) affects the access and operational decisions of a potential entrant (shipper  $E$ ).

In the same vein as Cremer *et Alii* [9], we point that the authority which regulates a dominant firm needs to take a sophisticated behaviour when access opportunities exist: she has to anticipate the ultimate market outcome resulting from the actions she delegates to the incumbent. This amounts to making her decisions hinging on *ex post* market realities. In the sequel of our analysis, when we characterize the optimal partial regulatory policy, we assume this to be the case, indeed.

In our maritime ferry sector, the outcome to be forecasted consists in a Stackelberg equilibrium, where access is encouraged and accommodated<sup>24</sup>. The regulator becomes herself a leader *vis-à-vis* the new operator and plays the first stage of the market game on behalf of the dominant firm she controls. Precisely as the public authority is assumed to be foresighted, so is the potential follower. To be more rigorous, the latter is persuaded that its actions will not trigger a reaction in the industry leader; in this sense, it is a "myopic" agent. Nevertheless, it bases its choices on the policy the regulator will impose once entry occurs. Consequently, if the sector is originally organized as a regulated monopoly and access subsequently occurs, then both the regulator and the entrant take the ultimate market outcome as the reference point of their decisional processes.

### 5.1 The Unregulated Entrant

As previously mentioned, we devote the present Section to investigate whether and under which circumstances firm  $E$  decides to enter our stylized shipping sector and, if so, how it selects prices and frequencies in its best interests, so that its profit function entails a maximum. It takes shipper  $I$ 's regulated actions as given and makes its own choices accordingly.

<sup>23</sup>Greece was granted a special exemption from full application of the Regulation until 2004, in consideration of the relevance of the inter-islands connections for the country.

<sup>24</sup>In Cremer *et Alii* [9] the outcome is, instead, a Nash-Cournot equilibrium.

Turning to the formal analysis, suppose that the pair of vectors  $(\mathbf{p}_I, \mathbf{f}_I)$  synthesizes the incumbent's actions. Given the latter, enterprise  $E$  finds it convenient to enter the market whenever there exist policies  $(\mathbf{p}_E, \mathbf{f}_E)$  such that

- a.  $(\mathbf{p}_E, \mathbf{f}_E) \neq (\mathbf{0}, \mathbf{0})$ , that is  $p_E^{s,i} > 0$  and  $f_E^s > 0$  for at least some  $i$  and  $s$ ;
- b.  $\pi_E(\mathbf{p}, \mathbf{f}) > 0$ , that is positive profits are generated<sup>25</sup>.

Intuitively enough, for the shipping activity to be undertaken, the associated return has to be at least as large as the one promised by the best outside opportunity, which is here normalized to zero. Depending on the market conditions, the firm may well decide to be active only in one season/segment, in the event that it would make smaller profits by doing otherwise<sup>26</sup>.

Conditionally on the favourable entry decision, shipper  $E$  sets  $\partial\pi_E/\partial p_E^{s,i} = 0$ ,  $\forall s, i$ , to select the (unique) price  $p_E^{s,i}$  at which the profit function entails a maximum<sup>27</sup>. This characterizes the reaction function  $p_E^{s,i}(p_I^{s,i}, f_I^s)$  that provides the optimal choice of  $p_E^{s,i}$  depending on the incumbent's price  $p_I^{s,i}$  and frequency  $f_I^s$ . Then firm  $E$ 's markup

$$\frac{p_E^{s,i} - a}{p_E^{s,i}} = \frac{1}{\varepsilon_E^{(s,i)(s,i)}}, \quad \forall s, i, \quad (30a)$$

is inversely proportional to the (absolute value of the) elasticity of demand  $X_E^{s,i}$  to price  $p_E^{s,i}$ , which is defined as  $\varepsilon_E^{(s,i)(s,i)} \equiv (p_E^{s,i}/X_E^{s,i}) (-\partial X_E^{s,i}/\partial p_E^{s,i})$ . Condition (30a) reveals that the shipper is more wary of the perverse impact of a high price on consumption, the more travellers react to a price increment by reducing their demand for the service. Provided that firm  $E$  clings on the inverse elasticity rule, it is, in fact, a monopolist *vis-à-vis* the market share it serves.

The first-order condition with respect to  $f_E^s$ , namely  $\partial\pi_E/\partial f_E^s = 0$ ,  $\forall s$ , characterizes the reaction function  $f_E^s(p_I^{s,i}, f_I^s)$ ,  $\forall i$ , that makes the optimal choice of  $f_E^s$  contingent on the incumbent's price  $p_I^{s,i}$ ,  $\forall i$ , and frequency  $f_I^s$  and yields

$$\sum_i (p_E^{s,i} - a) \frac{\partial X_E^{s,i}}{\partial f_E^s} = \phi_E, \quad \forall s. \quad (30b)$$

(30b) suggests that, at the firm's optimum, the variation induced by a frequency increase in the profit margins over all the marginal traffic units on both market segments must equal the cost of the last provided travel.

Combining (30a) and (30b), we further obtain the profit-maximizing number of connections for firm  $E$ , which is given by

$$f_E^{s,PR} = \frac{1}{\phi_E} \sum_i \frac{\eta_E^{(s,i)(s)}}{\varepsilon_E^{(s,i)(s,i)}} R_E^{s,i}, \quad \forall s, \quad (31)$$

where  $\eta_E^{(s,i)(s)} \equiv (f_E^s/X_E^{s,i}) (\partial X_E^{s,i}/\partial f_E^s)$  measures the elasticity of demand  $X_E^{s,i}$  to fre-

<sup>25</sup>In the text, we use the notation  $(\mathbf{p}, \mathbf{f})$  to represent the full vector of prices and frequencies of both shippers.

<sup>26</sup>Notice that, once firm  $E$  decides to operate in season  $s$ , it cannot refuse to serve one category of passengers and only accept the other. Nevertheless, a similar result can be achieved by properly adjusting the pricing policy, so that travellers belonging to the "unwanted" category only patronize the rival shipper.

<sup>27</sup>Both for prices and frequencies, uniqueness is ensured by the assumption of strict concavity of the profit functions. We as well suppose that the unique solution exists and is interior, so that choosing on the boundary of the feasible set of actions is suboptimal.

quency  $f_E^s$  and  $R_E^{s,i} \equiv X_E^{s,i} p_E^{s,i}$  the revenues obtained on segment  $i$  in season  $s$ . The condition above is interesting in that it identifies the relationship between the price elasticity and the frequency elasticity of demand at the entrant's optimum. One should first notice that, while the price elasticity of demand from category  $i$  refers to the price charged to the same category  $i$ , the frequency elasticity of demand from category  $i$  refers to the frequency provided to *both* categories of passengers. This follows from the event that connections cannot differentiate per market segment, whereas so can prices. One should as well observe that the number of connections is the ratio between a weighed sum of the revenues shipper  $E$  obtains from the tickets sold on the two market segments  $(R_E^{s,i})$ , the weights being the ratios between frequency and price elasticity for each segment  $(\eta_E^{(s,i)(s)} / \varepsilon_E^{(s,i)(s,i)})$ , and the cost of providing each connection by means of the available fleet in each season  $(\phi_E)$ .

## 5.2 The Impact of the Incumbent's Actions on the Entrant's Decisions: Propensity to Access and Strategic Relationships

By now, it should be clear that firm  $E$ 's choices crucially depend upon firm  $I$ 's actions. To fully understand the entrant's decisional process and the way it relates to the incumbent's behaviour, we hereafter investigate the impact of the latter on shipper  $E$ 's propensity to access the industry. Furthermore, we analyse the strategic nature of the relationship which arises between rival policies at the operational stage.

First of all, it is important to establish how reactive firm  $E$ 's profits are to shipper  $I$ 's prices. For this purpose, we differentiate  $\pi_E$  with respect to the rival price  $p_I^{s,i}$ , which yields

$$\frac{\partial \pi_E}{\partial p_I^{s,i}} = (p_E^{s,i} - a) \frac{\partial X_E^{s,i}}{\partial p_I^{s,i}}, \quad \forall s, i. \quad (32)$$

Since  $\partial X_E^{s,i} / \partial p_I^{s,i}$  is positive, so is  $\partial \pi_E / \partial p_I^{s,i}$ , provided that the margin  $(p_E^{s,i} - a)$  is larger than zero in its turn<sup>28</sup>. Therefore, the entrant's profits are (strictly) increasing in the rival price. This suggests that, *ceteris paribus*, the higher the monetary charge proposed by firm  $I$ , the larger the room for profitable entry by operator  $E$ . Said it differently, increments (resp., reductions) in the incumbent's prices have a positive (resp., negative) impact on the entrant's propensity to access the industry.

It is next relevant to understand which strategic relationship exists between rival prices. The latter shows up through the sign of the following derivative

$$\frac{\partial p_E^{s,i}}{\partial p_I^{s,i}} = - \frac{(p_E^{s,i} - a) \frac{\partial^2 X_E^{s,i}}{\partial p_E^{s,i} \partial p_I^{s,i}} + \frac{\partial X_E^{s,i}}{\partial p_I^{s,i}}}{(p_E^{s,i} - a) \frac{\partial^2 X_E^{s,i}}{\partial (p_E^{s,i})^2} + 2 \frac{\partial X_E^{s,i}}{\partial p_E^{s,i}}}, \quad \forall s, i^{29}. \quad (33)$$

The second-order condition of the profit maximization programme with respect to price imposes that the denominator of (33) be negative; therefore, we have

$$\text{sign} \left( \frac{\partial p_E^{s,i}}{\partial p_I^{s,i}} \right) = \text{sign} \left[ (p_E^{s,i} - a) \frac{\partial^2 X_E^{s,i}}{\partial p_E^{s,i} \partial p_I^{s,i}} + \frac{\partial X_E^{s,i}}{\partial p_I^{s,i}} \right], \quad \forall s, i.$$

<sup>28</sup>Since firm  $E$  can decide not to operate in unprofitable conditions, we take the margin to be, indeed, positive.

<sup>29</sup>(33) is obtained by differentiating the identity  $\partial \pi_E (p_E^{s,i}, f_E^s; p_I^{s,i}, f_I^s) / \partial p_E^{s,i} \equiv 0$ , which implicitly defines the entrant's reaction curve  $p_E^{s,i} (p_I^{s,i}, f_I^s)$ , with respect to  $p_I^{s,i}$  and then by solving for  $\partial p_E^{s,i} / \partial p_I^{s,i}$ .

As services are substitutes, the sign of the second term in the right-hand side is positive. On the other hand, reasonably enough, the cross partial derivative of  $X_E^{s,i}$  with respect to the rival price is either positive ( $\partial^2 X_E^{s,i} / \partial p_E^{s,i} \partial p_I^{s,i} > 0$ ) or not too negative ( $\partial^2 X_E^{s,i} / \partial p_E^{s,i} \partial p_I^{s,i} < 0$  and  $|\partial^2 X_E^{s,i} / \partial p_E^{s,i} \partial p_I^{s,i}|$  small). This means that, as the rival commodity gets more expensive, the decrement that is induced in  $X_E^{s,i}$  by an increase in  $p_E^{s,i}$  becomes less important. Hence, we can conclude that firm  $E$ 's reaction curve is upward sloping, so that it is  $\partial p_E^{s,i} / \partial p_I^{s,i} > 0$ . This involves that the incumbent's prices are *strategic complements* for shipper  $E$  : the higher (resp., lower) the monetary charge proposed by the incumbent, the higher (resp., lower) the price the entrant can set in its turn.

We now turn to the impact induced by a variation in the incumbent's frequency on firm  $E$ 's entry decision. Differentiating  $\pi_E$  with respect to  $f_I^s$  yields

$$\frac{\partial \pi_E}{\partial f_I^s} = \sum_i (p_E^{s,i} - a) \frac{\partial X_E^{s,i}}{\partial f_I^s}, \quad \forall s. \quad (34)$$

The demand  $X_E^{s,i}$  decreases in the amount of connections offered by the dominant firm. Therefore, with positive margins, one has  $\partial \pi_E / \partial f_I^s < 0$  : all else equal, the entrant's profits are a decreasing function of the rival number of travels. This involves that the more (resp., fewer) travels shipper  $I$  operates, the less (resp., more) attractive entry is to the new operator.

Finally, we need to investigate the impact of the incumbent's scheduling on the entrant's marginal profits. This is characterized by the sign of the derivative

$$\frac{\partial f_E^s}{\partial f_I^s} = - \frac{\sum_i (p_E^{s,i} - a) \frac{\partial^2 X_E^{s,i}}{\partial f_E^s \partial f_I^s}}{\sum_i (p_E^{s,i} - a) \frac{\partial^2 X_E^{s,i}}{\partial (f_E^s)^2}}, \quad \forall s^{30}. \quad (35)$$

Once again, the denominator in (35) is negative by the second-order condition of the profit maximization problem with respect to frequency. Therefore, one has

$$\text{sign} \left( \frac{\partial f_E^s}{\partial f_I^s} \right) = \text{sign} \left[ \sum_i (p_E^{s,i} - a) \frac{\partial^2 X_E^{s,i}}{\partial f_E^s \partial f_I^s} \right], \quad \forall s.$$

The previous equality reveals that the strategic relationship between rival frequencies ultimately depends on how firm  $E$ 's marginal demand reacts to increases in the number of connections operated by the opponent. We expect the cross-partial derivative  $X_E^{s,i}$  with respect to the rival frequency to be negative ( $\partial^2 X_E^{s,i} / \partial f_E^s \partial f_I^s < 0$ ), meaning that an improvement in the quality of the rival product reduces the growth that is caused in  $X_E^{s,i}$  by adding own connections. It follows that  $\partial f_E^s / \partial f_I^s$  is negative, that is the incumbent's frequencies are *strategic substitutes* for operator  $E$ . *Ceteris paribus*, the more numerous (resp., fewer) the connections supplied by the dominant shipper, the fewer (resp., the more) the ones the opponent operates. Hence, when the incumbent offers many travels to the population of passengers, to some extent, the entrant gets crowded out.

It is interesting to parallel this result with the one Cremer *et Alii* [7] obtain in their vertical differentiation model about the postal sector. They find that firms' best-reply functions in the quality game are upward sloping, meaning that qualities are strategic

<sup>30</sup>Similarly to (33), (35) is obtained by differentiating the identity  $\partial \pi_E (p_E^{s,i}, f_E^s; p_I^{s,i}, f_I^s) / \partial f_E^s \equiv 0$ , which implicitly defines the entrant's reaction curve  $f_E^s (p_I^{s,i}, f_I^s)$ , with respect to  $f_I^s$  and then by solving for  $\partial f_E^s / \partial f_I^s$ .

complements. We have just seen that this is not the case in our framework. Nevertheless, the circumstance that the larger the frequency provided by either shipper, the smaller the rival's, should be viewed as a way competitors differentiate their services. This is similar to having firms choose different qualities to soften price competition, a typical effect of vertical differentiation models.

We conclude by stating the following Proposition, which summarizes the results we have achieved in this Section.

**Proposition 2** *In the shipping industry, as long as aggregate demands satisfy some reasonable properties, firm E's propensity to entry increases in firm I's prices and decreases in firm I's amount of connections. Moreover, since the incumbent's prices are strategic complements and the incumbent's frequencies strategic substitutes for shipper E, the entrant's marginal profits augment in the incumbent's prices and reduce in the incumbent's amount of connections.*

### 5.3 The Optimal Price-and-Frequency Policy

Once the regulator is aware of the effects firm I's actions induce on shipper E's decisions (as synthesized in Proposition 2), she can properly design the partial regulatory regime. One should recall that this amounts to directly shaping the incumbent's market behaviour, whereas the entrant operates as an unregulated profit-maximizer.

In formal terms, the regulator characterizes the prices and frequencies  $(\mathbf{p}_I^{PR}, \mathbf{f}_I^{PR})$  which solve the utilitarian social programme

$$\begin{aligned} \underset{\{\mathbf{p}_I^{s,i}, \mathbf{f}_I^s\}_{\forall s,i}}{\text{Max}} \quad & V(\mathbf{p}, \mathbf{f}; \boldsymbol{\alpha}, \boldsymbol{\tau}) + \pi_I(\mathbf{p}_I, \mathbf{f}_I) + \pi_E(\mathbf{p}_E^{PR}(\mathbf{p}_I, \mathbf{f}_I), \mathbf{f}_E^{PR}(\mathbf{p}_I, \mathbf{f}_I)) \\ \text{subject to} \quad & \pi_I(\mathbf{p}_I, \mathbf{f}_I) \geq 0. \end{aligned} \quad (36)$$

In (36), the objective function is the unweighted sum of consumer surplus and firms' profits. Moreover,  $\mathbf{p}_E^{PR}(\mathbf{p}_I, \mathbf{f}_I)$  and  $\mathbf{f}_E^{PR}(\mathbf{p}_I, \mathbf{f}_I)$  are the vectors of contingent choices the entrant performs, clinging on the optimal private rules (30a) and (30b). The superscript *PR* stays for *partial regulation*. As under monopoly regulation, the dominant shipper's budget is secured.

The optimal prices and frequencies respectively satisfy the first-order conditions

$$\frac{d\pi_I}{dp_I^{s,i}} = \left( -\frac{\partial V^{s,i}}{\partial p_I^{s,i}} - \frac{\partial \pi_E}{\partial p_I^{s,i}} \right) \frac{1}{1 + \lambda^{PR}}, \quad \forall s, i \quad (37a)$$

and

$$-\frac{d\pi_I}{df_I^s} = \left( \frac{\partial V^s}{\partial f_I^s} + \frac{\partial \pi_E}{\partial f_I^s} \right) \frac{1}{1 + \lambda^{PR}}, \quad \forall s, \quad (37b)$$

where  $\lambda^{PR}$  is the shadow cost associated with the break-even constraint when shipper I is subject to partial regulation. As compared to (23a) and (23b), (37a) and (37b) display two major changes, which we hereafter illustrate.

Firstly, the left-hand sides contain the (absolute values of the) *total*, rather than the partial, derivatives of profits  $\pi_I$  with respect to price  $p_I^{s,i}$  and to frequency  $f_I^s$ , namely

$$\frac{d\pi_I}{dp_I^{s,i}} = \frac{\partial \pi_I}{\partial p_I^{s,i}} + \frac{\partial \pi_I}{\partial p_E^{s,i}} \frac{\partial p_E^{s,i}}{\partial p_I^{s,i}} + \frac{\partial \pi_I}{\partial f_E^s} \frac{\partial f_E^s}{\partial p_I^{s,i}}$$

and

$$\frac{d\pi_I}{df_I^s} = \frac{\partial\pi_I}{\partial f_I^s} + \frac{\partial\pi_I}{\partial f_E^s} \frac{\partial f_E^s}{\partial f_I^s} + \frac{\partial\pi_I}{\partial p_E^{s,i}} \frac{\partial p_E^{s,i}}{\partial f_I^s}$$

respectively. The above expressions show that the variation that is induced by a price (resp., frequency) change in the incumbent's profits consists in three effects. To begin with, there is a direct effect, which operates through the own price (resp., frequency). Additionally, two indirect effects can be identified, which work through the rival price and the rival frequency. The former prevails over each of the latter.

Secondly, the right-hand sides of (37a) and (37b) include the marginal effect of the incumbent's actions not only on consumer surplus ( $-\partial V^{s,i}/\partial p_I^{s,i} = X_I^{s,i}$  and  $\partial V^s/\partial f_I^s$  respectively), but also on the rival profits ( $-\partial\pi_E/\partial p_I^{s,i}$  and  $\partial\pi_E/\partial f_I^s$  respectively). This means that partial regulation forces the targeted firm to "more comprehensive" preoccupations than so does monopoly regulation. At the same time, partial regulation imposes less stringent obligations, as suggested by the signs of the terms under scrutiny<sup>31</sup>. To make the point about concern comprehensiveness more evident, we manipulate (37a) and (37b) and get the equality

$$\frac{d\pi_I/df_I^s}{d\pi_I/dp_I^{s,i}} = \frac{\partial(V^s + \pi_E)/\partial f_I^s}{\partial(V^s + \pi_E)/\partial p_I^{s,i}}, \quad \forall s, i. \quad (38)$$

As under monopoly, the left-hand side of (38) is the rate at which the regulated prices and frequencies can be substituted away at zero shipper's profits, except that now it embodies the indirect impact of the controlled variables through the entrant's. Instead, the right-hand side differs from the monopoly case: it expresses the substitution rate as for consumer surplus *together with* rival profits, that is as for the benefits of all economic agents but the regulated shipper<sup>32</sup>.

The essential point to be retained is that the incumbent's prices and frequencies are optimally chosen by the regulator so that passengers are encouraged to travel with the entrant to the extent that it is efficient to do so. Said it differently, the dominant shipper's prices and number of connections are determined so that the public sector does not crowd out the unregulated operator, as long as the latter is not inefficient. Recall that shipper  $E$ 's unit cost of connections is larger than shipper  $I$ 's ( $\phi_E > \phi_I$ ); on the other hand, the unit cost firm  $E$  bears in terms of traffic volume is smaller than the one of firm  $I$  ( $a < a + \gamma$ ). Due to this circumstance, allocating passengers suitably between shippers constitutes a delicate task; in particular, it requires more caution than it would in a duopoly where the quality dimension did not matter. Indeed, in that case, the entrant would produce a positive output, at equilibrium, only if the dominant firm bears an unambiguous cost disadvantage<sup>33</sup>.

Remarkably, having shipper  $E$  enter the industry may sometimes help relax the regulated operator's budget constraint, because the regulator is less requiring. The way this occurs shows up as soon as one studies the case in which no budget concern arises. This is a limit scenario, but it helps intuition. Imposing  $\lambda^{PR} = 0$ , (37a) rewrites as

$$\left[ p_I^{s,i} - (a + \gamma) \right] \left| \frac{dX_I^{s,i}}{dp_I^{s,i}} \right| = (p_E^{s,i} - a) \frac{\partial X_E^{s,i}}{\partial p_I^{s,i}}, \quad \forall s, i. \quad (39)$$

The margin  $\left[ p_I^{s,i} - (a + \gamma) \right]$  in the left-hand side of (39) measures the distortion associated

<sup>31</sup>For instance, in (37b) we have  $\partial V^s/\partial f_I^s > 0$  but  $\partial\pi_E/\partial f_I^s < 0$ .

<sup>32</sup>Recall that, as already pointed in a previous footnote, under our assumptions about the demand side of the shipping market, it is  $\partial V^{s,i}/\partial p_I^{s,i} = \partial V^s/\partial p_I^{s,i}, \forall s, i$ .

<sup>33</sup>See Estrin and de Meza [12], who prove this result for a mixed oligopoly in which competition occurs between State-owned and private firms.

with the (absolute value of the) variation induced by a unit increase in the regulated price in shipper  $I$ 's demand  $(dX_I^{s,i})$ . The margin  $(p_E^{s,i} - a)$  in the right-hand side is, instead, the distortion associated with the variation caused by the same price increase in firm  $E$ 's demand  $(\partial X_E^{s,i})$ . For the purpose of minimizing the two distortions, the regulator has to account for shipper  $E$ 's margin and concede a positive margin to the regulated firm as well. Therefore, under partial regulation, the unregulated entrant makes positive profits, even if the regulator is perfectly informed about all relevant conditions. Moreover, in the absence of break-even preoccupations, also the regulated incumbent is given up a positive margin, through which it fully covers its costs.

Notice however that, if both shippers operate at zero margins, then the "mixed" equilibrium characterized by the condition in (39), where the budget constraint does not bind, yields the socially optimal two-providers allocation. This result recalls the one achieved by Cremer *et Alii* [7], who nevertheless model the public and the private firm in the postal sector as Nash-competitors, rather than Stackelberg players.

Furthermore, the left-hand side of (39) is null, hence so is the right-hand side, in the absence of a direct price effect on the regulated shipper's demand  $(\partial X_I^{s,i}/\partial p_I^{s,i} = 0)$ . In this event, firm  $I$ 's demand is completely inelastic to the own price. Therefore, price is not a concern. All that matters is passenger allotment between firms, hence frequency, and the socially optimal allocation is again achievable.

Finally, if services are unrelated, then firm  $E$ 's profits are insensitive to variations in the leader's price  $(\partial X_E^{s,i}/\partial p_I^{s,i} = 0)$  and the right-hand side of (39) is null. This suggests that, in the ideal world without budget preoccupations, service substitutability is beneficial to both shippers. *Ceteris paribus*, as long as the cross-price effect  $\partial X_E^{s,i}/\partial p_I^{s,i}$  is important, the negative effect of a price increment on firm  $I$ 's traffic volume is partly compensated by the positive impact on the entrant's demand, which reduces the extent to which the regulated price should be increased. Furthermore, since prices are strategic complements, this also prevents the rival price from growing excessively.

The more realistic case for  $\lambda^{PR} > 0$  requires that, *ceteris paribus*, the incumbent's margins be larger, because the budget constraint is now more stringent. In particular, with  $\lambda^{PR} > 0$ , (37a) rewrites as

$$\frac{p_I^{s,i} - (a + \gamma)}{p_I^{s,i}} = \frac{\varepsilon_{IE}^{(s,i)(s,i)}/\varepsilon_E^{(s,i)(s,i)} + \lambda^{PR}}{\widehat{\varepsilon}_I^{(s,i)(s,i)}(1 + \lambda^{PR})}, \quad \forall s, i, \quad (40a)$$

where  $\widehat{\varepsilon}_I^{(s,i)(s,i)}$  is the elasticity of demand  $X_I^{s,i}$  to price  $p_I^{s,i}$  adjusted for the strategic effect and  $\varepsilon_{IE}^{(s,i)(s,i)}$  is the cross-elasticity of demand  $X_I^{s,i}$  to the rival price  $p_E^{s,i}$  (details are relegated to Appendix C). (40a) reveals that the regulated markup on market segment  $i$  in season  $s$  is inversely proportional to the elasticity of demand  $X_I^{s,i}$  to own price  $p_I^{s,i}$ , adjusted for the strategic effect between firms. Moreover, it is discounted for the shadow cost of the budget constraint. These results are similar to those we established for the regulated monopolist. Yet, in the scenario under scrutiny, something new appears. The markup is now directly proportional to a term which depends on the additional price elasticities that become relevant as soon as another provider is active in the sector. In particular, it is increasing in the elasticity of the incumbent's demand to the rival price as well as in the shadow cost of the budget constraint; it is instead decreasing in the own-price elasticity of the entrant's demand.

In its turn, (37b) can be manipulated till the optimal number of connections for the regulated shipper is found to be given by (see Appendix C for the derivation of the following

expression)

$$f_I^{s,PR} = \frac{\sum_i \hat{\eta}_I^{(s,i)(s)} \left( \frac{\varepsilon_{IE}^{(s,i)(s,i)} / \varepsilon_E^{(s,i)(s,i)} + \lambda^{PR}}{\hat{\varepsilon}_I^{(s,i)(s,i)}} \right) R_I^{s,i} - \sum_i \frac{\eta_{EI}^{(s,i)(s)}}{\varepsilon_E^{(s,i)(s,i)}} R_E^{s,i}}{(1 + \lambda^{PR}) \phi_I - \frac{\partial V^s}{\partial f_I^s}}, \quad (40b)$$

In (40b),  $\hat{\eta}_I^{(s,i)(s)} \equiv (f_I^s / X_I^{s,i}) (dX_I^{s,i} / df_I^s)$  indicates the elasticity of demand  $X_I^{s,i}$  to frequency  $f_I^s$  adjusted to account for the entrant's reaction,  $R_I^{s,i} \equiv X_I^{s,i} p_I^{s,i}$  expresses the revenues firm  $I$  obtains on segment  $i$  in season  $s$  and  $\eta_{EI}^{(s,i)(s)} \equiv (f_I^s / X_E^{s,i}) (-\partial X_E^{s,i} / \partial f_I^s)$  measures the (absolute value of the) elasticity of demand  $X_E^{s,i}$  to rival frequency  $f_I^s$ . We hereafter discuss (40b) in details.

To begin with, one should notice that, *ceteris paribus*,  $f_I^{s,PR}$  is larger, the larger the benefit generated for the passengers by firm  $I$ 's last connection ( $\partial V^s / \partial f_I^s$ ). On the opposite,  $f_I^{s,PR}$  decreases in the marginal cost of connections ( $\phi_I$ ). These conclusions immediately follow from the denominator of (40b).

Furthermore, all else equal,  $f_I^{s,PR}$  increases in the weighed sum of the incumbent's revenues  $\sum_i \frac{\hat{\eta}_I^{(s,i)(s)}}{\hat{\varepsilon}_I^{(s,i)(s,i)}} R_I^{s,i} \left( \frac{\varepsilon_{IE}^{(s,i)(s,i)}}{\varepsilon_E^{(s,i)(s,i)}} + \lambda^{PR} \right)$ . Let us analyse this expression. First recall that shipper  $I$ 's markup on each market segment is directly proportional to the ratio  $\left( \frac{\varepsilon_{IE}^{(s,i)(s,i)} / \varepsilon_E^{(s,i)(s,i)} + \lambda^{PR}}{\hat{\varepsilon}_I^{(s,i)(s,i)}} \right)$ . This suggests an intuitive conclusion, namely that, *ceteris paribus*, when the regulated operator is attributed a larger relative margin, it is also required to provide more connections. Moreover, the weighed sum under scrutiny, hence  $f_I^{s,PR}$ , increases in the own-frequency elasticity of demand; this means that, all else equal, the more elastic demand is to scheduling, the larger the number of connections to be provided, which is again quite intuitive.

Frequency  $f_I^{s,PR}$  decreases in the weighed sum of rival's revenues  $\sum_i \frac{\eta_{EI}^{(s,i)(s)}}{\varepsilon_E^{(s,i)(s,i)}} R_E^{s,i}$ . *Ceteris paribus*, the larger the price elasticity  $\varepsilon_E^{(s,i)(s,i)}$ , hence the smaller the entrant's markup, the more travels the regulated firm operates. To see why this is the case, consider that when firm  $E$  obtains small markup, its price is not too high, then nor so is firm  $I$ 's. Therefore, the regulated shipper is asked to operate an important amount of travels without the need to allow for too high a tariff. This contributes to contain the raise in the average market price. On the other hand, the more elastic the entrant's demand is to  $f_I^s$ , the fewer connections firm  $I$  can be required to offer. This prevents the rival's demand from significantly shrinking.

One should as well observe that the optimal number of connections under partial regulation depends on the shadow cost of the budget constraint. On one side,  $f_I^{s,PR}$  increases in  $\lambda^{PR}$  because the latter is larger when higher prices are fixed for the regulated shipper. On the other side,  $f_I^{s,PR}$  is to be deflated by more than  $\lambda^{PR}$ , as the denominator of (40b) shows. Indeed, further scheduling makes budget balance more costly, which translates into a larger value of  $\lambda^{PR}$ . From the analysis so far performed, it should be clear that entry contributes to relax the budget constraint if it helps lessen the regulatory requirements.

On the basis of what we have so far illustrated, we are finally able to identify two limit cases. The first one is represented by  $f_I^{s,PR} = 0$ . For this scenario to arise, one needs to have  $\sum_i \frac{\hat{\eta}_I^{(s,i)(s)}}{\hat{\varepsilon}_I^{(s,i)(s,i)}} R_I^{s,i} \left( \frac{\varepsilon_{IE}^{(s,i)(s,i)}}{\varepsilon_E^{(s,i)(s,i)}} + \lambda^{PR} \right) = \sum_i \frac{\eta_{EI}^{(s,i)(s)}}{\varepsilon_E^{(s,i)(s,i)}} R_E^{s,i}$ . Said it in words, shipper  $I$  is required not to operate at all whenever the weighed sum of its revenues equals the one of firm  $E$ 's. The entrant is then let replace the regulated incumbent and a new monopoly arises.

The second limit situation is the one for  $f_I^{s,PR} \rightarrow \infty$ . This is the case when the denominator of the right-hand side of (40b) is null, that is when we have  $(1 + \lambda^{PR}) \phi_I = \partial V^s / \partial f_I^s$ . This means that shipper  $I$  should schedule infinitely, as long as the marginal social cost of the last provided connection, namely  $(1 + \lambda^{PR}) \phi_I$ , is fully compensated by the marginal benefit it generates for passengers  $(\partial V^s / \partial f_I^s)$ .

To conclude the analysis, we state the following Proposition, which collects the main results of the present Section.

**Proposition 3** *Under partial regulation, the unregulated shipper pockets a rent even in a complete information environment. On the contrary, with  $\lambda^{PR} > 0$ , the regulated firm always obtains zero profits. Yet entry may help achieve budget balance if it allows the regulator to be less requiring vis-à-vis the dominant operator.*

#### 5.4 Decentralization through a Global Price-and-Frequency Constraint

In this Section, we show how the quality-adjusted price cap proposed by De Fraja and Iozzi [10] should be modified for the policy  $(\mathbf{p}_I^{PR}, \mathbf{f}_I^{PR})$  to be decentralized to a profit-maximizing operator, which (eventually) competes as a market leader with an unregulated follower.

Formally speaking, in the scenario under scrutiny, firm  $I$  is required to meet a constraint that is similar to the one in (27), so that its programme writes as

$$\begin{aligned} & \underset{\{p_I^{s,i}, f_I^s\}_{\forall s,i}}{\text{Max}} \quad \pi_I(\mathbf{p}_I, \mathbf{f}_I) \\ & \text{subject to} \\ & \sum_{s,i} \beta_{DPR}^{s,i} p_I^{s,i} - \sum_s \alpha_{DPR}^s f_I^s \leq P_{DPR}, \end{aligned} \tag{41}$$

where the script  $DPR$  means *decentralized partial regulation*. The interpretation is exactly the same as the one we illustrated for (27) and we do not repeat it here. The first-order conditions of (41) with respect to prices and frequencies are respectively given by

$$\frac{d\pi_I}{dp_I^{s,i}} = \lambda^{DPR} \beta_{DPR}^{s,i}, \quad \forall s, i \tag{42a}$$

and

$$-\frac{d\pi_I}{df_I^s} = \lambda^{DPR} \alpha_{DPR}^s, \quad \forall s, \tag{42b}$$

$\lambda^{DPR}$  being the Lagrange multiplier associated with the regulatory constraint in (41). If the regulator wants the previous conditions to hold for the appropriate value of  $\lambda^{DPR}$ , she needs to make sure that the equalities

$$\lambda^{DPR} = \frac{1}{1 + \lambda^{PR}}, \tag{43a}$$

$$\beta_{DPR}^{s,i} = -\frac{\partial V^{s,i,PR}}{\partial p_I^{s,i}} - \frac{\partial \pi_E^{PR}}{\partial p_I^{s,i}}, \quad \forall s, i \tag{43b}$$

and

$$\alpha_{DPR}^s = \frac{\partial V^{s,PR}}{\partial f_I^s} + \frac{\partial \pi_E^{PR}}{\partial f_I^s}, \quad \forall s \tag{43c}$$

are simultaneously satisfied. Observe that the superscript  $PR$  is used to mean that the functions are evaluated at the solution  $(\mathbf{p}_I^{PR}, \mathbf{f}_I^{PR})$ .

Manipulating the derivatives of firm  $E'$ 's profits with respect to  $p_I^{s,i}$  and to  $f_I^s$ , (43b) and (43c) respectively become

$$\beta_{DPR}^{s,i} = -\underbrace{\frac{\partial V^{s,i,PR}}{\partial p_I^{s,i}}}_{X_I^{s,i,PR}} + X_E^{s,i,PR} \underbrace{\left( \frac{\partial X_E^{s,i,PR}}{\partial X_E^{s,i,PR}} / \frac{\partial p_I^{s,i}}{\partial p_E^{s,i}} \right)}_{<0}, \quad \forall s, i, \quad (44a)$$

and

$$\alpha_{DPR}^s = \frac{\partial V^{s,PR}}{\partial f_I^s} + \sum_i X_E^{s,i,PR} \underbrace{\left( \frac{\partial X_E^{s,i,PR}}{\partial f_I^s} / \frac{-\partial X_E^{s,i,PR}}{\partial p_E^{s,i}} \right)}_{<0}, \quad \forall s^{34}. \quad (44b)$$

Expressions (44a) and (44b) are quite instructive. In what follows, we investigate each of them in details.

Let us begin with (44a). It reveals that weight  $\beta_{DPR}^{s,i}$  is given by the difference between two terms. The first term is the marginal passenger surplus generated as the regulated price varies, namely the incumbent's demand evaluated at the partial regulation prices and frequencies  $(X_I^{s,i,PR})$ . This is just the same as for a regulated monopoly. The second term, which appears under partial regulation of duopoly, consists in the demand faced by shipper  $E$ , evaluated at the partial regulation solution  $(X_E^{s,i,PR})$ , times the marginal rate of substitution between rival prices  $\left( \frac{\partial X_E^{s,i,PR}}{\partial p_I^{s,i}} / \frac{\partial X_E^{s,i,PR}}{\partial p_E^{s,i}} \right)$ . Notice that we have  $\partial X_E^{s,i,PR} / \partial p_I^{s,i} < |\partial X_E^{s,i,PR} / \partial p_E^{s,i}|$  because the cross-price effect on demand is less important than the own-price effect. Recall as well that it is  $\partial X_E^{s,i,PR} / \partial p_I^{s,i} > 0$  and  $\partial X_E^{s,i,PR} / \partial p_E^{s,i} < 0$ . It follows that firm  $E$ 's demand is attributed a weight which is negative and smaller than unity, whereas the incumbent's a weight that is positive and equal to one. In definitive, the appropriate price weight  $\beta_{DPR}^{s,i}$  in the decentralization scheme is given by the difference between the regulated operator's demand and a portion of the unregulated provider's<sup>35</sup>.

Let us further analyze the impact of the single components on the price weight. *Ceteris paribus*, the larger the entrant's demand, the smaller  $\beta_{DPR}^{s,i}$ . Under this circumstance, neither the incumbent's price nor the rival's is too low. This suggests that when the traffic served by the entrant is large, price competition between shippers is soft, that is the regulated incumbent is favoured by the access of an "important" provider because this helps relax the price constraint.

Furthermore, *ceteris paribus*,  $\beta_{DPR}^{s,i}$  is smaller, the bigger the cross-price effect on firm  $E$ 's demand. If  $X_E^{s,i,PR}$  is very reactive to the incumbent's price, then the demand significantly increases as  $p_I^{s,i}$  is raised. In this event, the variation in shipper  $E$ 's demand compensates the reduction in firm  $I$ 's by an important amount. Once again, neither price needs be very low, i.e. price competition is soft when the entrant's demand is particularly "sensitive" to the regulated price.

Finally, *ceteris paribus*,  $\beta_{DPR}^{s,i}$  is smaller, the less important the (absolute value of the) own-price effect on shipper  $E$ 's demand. If the latter does not vary significantly as  $p_E^{s,i}$  gets higher, price competition is softened and rival prices remain both relatively large.

Let us next devote attention to (44b). Also weight  $\alpha_{DPR}^s$  is given by the difference

<sup>34</sup>More precisely, the second term in (44a) and (44b) is obtained by using the first-order condition of shipper  $E$ 's profit-maximization programme with respect to price  $p_E^{s,i}$ , which yields  $(p_E^{s,i} - a) = -X_E^{s,i} / (\partial X_E^{s,i} / \partial p_E^{s,i})$ .

<sup>35</sup>In turn, this implies that the price weight could only be null if the entrant served larger traffic than the incumbent at the partial regulation prices and frequencies  $(X_E^{s,i,PR} > X_I^{s,i,PR})$ . In this limit case, firm  $I$  would no longer be the market leader and a price constraint would not actually be imposed.

between two terms. The first term is the marginal passenger surplus generated by a variation in the incumbent's number of connections. Analogous term was found to be present when a monopolistic industry was under scrutiny. The second term shows up as soon as a partially regulated duopoly is at stake. It is a weighed sum of the demands faced by firm  $E$  in the two market segments. In this sum, weights are given by the marginal rates of substitution between own prices and rival frequency  $\left( \frac{\partial X_E^{s,i,PR}/\partial f_I^s}{-\partial X_E^{s,i,PR}/\partial p_E^{s,i}} \right)$ . Recalling

that we have  $\partial X_E^{s,i,PR}/\partial f_I^s < 0$  together with  $\partial X_E^{s,i,PR}/\partial p_E^{s,i} < 0$ , we deduce that the weights assigned to shipper  $E$ 's demands are negative. On the other hand, without deeper knowledge of the characteristics of the entrant's traffic, we cannot precisely establish the size of those weights. We can nevertheless identify an upper bound for their magnitude. For this purpose, we rewrite the rates under scrutiny as the products of two other rates, that is as

$$\frac{\partial X_E^{s,i,PR}/\partial f_I^s}{-\partial X_E^{s,i,PR}/\partial p_E^{s,i}} = \left( \frac{\partial X_E^{s,i,PR}/\partial f_I^s}{\partial X_E^{s,i,PR}/\partial f_E^s} \right) \left( \frac{\partial X_E^{s,i,PR}/\partial f_E^s}{-\partial X_E^{s,i,PR}/\partial p_E^{s,i}} \right), \forall s, i.$$

The first term of the product, namely  $\frac{\partial X_E^{s,i,PR}/\partial f_I^s}{\partial X_E^{s,i,PR}/\partial f_E^s}$ , measures the marginal rate of substitution between own and rival frequency for shipper  $E$ . This rate is negative and smaller than unity (in absolute value) because the cross-scheduling effect exceeds the own effect.

The second term in the product, namely  $\frac{\partial X_E^{s,i,PR}/\partial f_E^s}{-\partial X_E^{s,i,PR}/\partial p_E^{s,i}}$ , expresses the (absolute value of the) marginal rate of substitution between own price and own frequency for firm  $E$ . Its magnitude cannot be established. Yet combining our observations about the two terms, we conclude that the weights attributed to provider  $E$ 's demands, as appearing in the right-hand side of (44b), cannot exceed (and, indeed, are smaller than) the marginal rates of substitution between own prices and own frequency.

Notice that, *ceteris paribus*, the larger shipper  $E$ 's demands at the partial regulation prices and frequencies, the smaller  $\alpha_{DPR}^s$ . As subject to a sufficiently soft frequency constraint, firm  $I$  provides not too large a number of connections. Since shipper  $I$ 's frequencies are strategic substitutes for provider  $E$ 's, the entrant operates a significant amount of travels and is not crowded out by the regulation. In definitive, whenever entry is "important", the regulator is induced to be less requiring with the incumbent not only in terms of pricing, but also in terms of scheduling. It is then easier to ensure the regulated operator's viability.

Furthermore, all else equal, the larger the (absolute value of the) cross-frequency effect on the entrant's demand, the smaller  $\alpha_{DPR}^s$ . This means that when the entrant's demand significantly changes as the incumbent's frequency increases, the regulator is not particularly stringent with shipper  $I$ . If she were so, then she would crowd out the competitor, whose demand is very reactive to firm  $I$ 's scheduling.

Finally, *ceteris paribus*, the smaller the (absolute value of the) own-price effect on the entrant's demand, the smaller  $\alpha_{DPR}^s$ . As previously mentioned, if demand does not vary significantly as  $p_E^{s,i}$  gets higher, then not too low prices are charged. When  $p_E^{s,i}$  is sufficiently high, the entrant can (and has an incentive to) provide an important amount of connections. Under such a circumstance, it is not necessary to be very requiring in terms of scheduling with the regulated operator.

Our analysis of (44a) and (44b) suggests that the smaller the price weight  $\beta_{DPR}^{s,i}$ , the higher the average price in the shipping sector, which is detrimental for passengers. On the other hand, the smaller the frequency weight  $\alpha_{DPR}^s$ , the fewer the connections operated by the incumbent, but the more the ones provided by the entrant. According to (35), it is  $|\partial f_E^s/\partial f_I^s| < 1$ , meaning that as  $f_I^s$  reduces by one,  $f_E^s$  increases by more than one, so that the effect of total frequency is positive. As a conclusion, the net impact on the

surplus of the whole population of travellers critically depends on the price-scheduling balance. As for the specific passenger types, those who particularly care about tariffs (type-1 customers) might be made worse off, whereas the more impatient ones (type-2 customers) may well benefit from more intensive scheduling.

Our investigation of (44a) and (44b) also conveys an essential instruction to the regulatory body of partially regulated industries: despite partial regulation does not directly concern the eventual entrants, decentralization of the optimal policy to the dominant firm should be based also on the traffic served by the unregulated competitors. It should as well account for the reactivity of the competitors' demands to each of the relevant variables. In particular, (44a) suggests how to apply the price-cap method to partially regulated environments. This represents a novelty in the economic literature; indeed, pricing decentralization has been widely explored in regulated monopoly settings, not in the presence of unregulated strategic rivals.

A non-negligible implication of the above conclusion is that the authority should be allowed to use the available knowledge (if any) and/or to extract information (otherwise) about the overall industry. This might posit practical difficulties in contexts where regulatory bodies are restricted to solely use information about the targeted operators (if available). Nevertheless, in real-world situations, restrictions are more often imposed as to the usage of information concerning other markets, rather than competing operators in the same regulated market<sup>36</sup>. As long as this is the case, regulators face no additional difficulties than those arising from standard information eliciting.

## 6 Addressing Concerns about Redistribution: The Territorial Continuity Principle (Very Preliminary)

In their work about optimal pricing in the postal sector, Billette de Villemeur *et Alii* [4] raise the observation that both the optimal solution and the decentralization scheme are likely to significantly change, if the social planner also points to redistributive objectives. This issue acquires prominent importance as far as the maritime ferry industry is concerned. This is so because society believes that the drawbacks associated with the physical disconnection of the islands from the mainland should be limited and the penalized people compensated for those disadvantages by means of sufficiently favourable transport conditions. Such a value judgement is embodied in the *universal service principle* or, better, in its specification as *territorial continuity principle*.

In the same vein as Billette de Villemeur *et Alii* [4], we need to stress that the policies so far characterized may fail to guarantee that a reasonable level of territorial continuity be achieved. To see this, consider the low season: during this period, traffic is scarce and essentially composed by islanders. Given the limited size of the demand, it may prove suboptimal, on pure efficiency grounds, to require the shipper to provide as large a number of travels as it would be satisfactory from different perspectives. On the other hand, as residential customers are highly captive and the regulated tariffs depend on the price elasticity of demand, efficiency criteria can make transportation services hardly affordable precisely to those who most need to travel<sup>37</sup>.

For regulatory policies supporting a specific interest group (namely, the islanders) to be drawn, it is necessary to amend the social planner's programme, so that a weight larger than unity is attributed to the islanders in the welfare function<sup>38</sup>. In this pro-residents

<sup>36</sup>In general, yardstick competition mechanisms precisely hinge on the fact that information revealed by different agents is plaid against one another for the purposes of performance improvement and rent extraction.

<sup>37</sup>The most immediate way to realize this is to recall the Ramsey-Boiteux formula in (25).

<sup>38</sup>As Martimort [21] stresses, this formulation is the one used by Shapiro and Willig (1990) to model a biased political principal. Following these Authors, in an asymmetric information environment, Martimort

world, the optimal (constrained) policies under monopoly regulation and duopoly partial regulation are then characterized by proceeding exactly as in the previous Sections.

The duties the regulated provider bears when redistribution is an issue are *territorial continuity* (or *public service*) *obligations*. Not so are, instead, the regulatory requirements imposed for pure efficiency purposes. A similar point is made by Cremer *et Alii* [7] for the provision of postal services; these Authors stress that universal service constraints (the analogous of the territorial continuity obligations in the postal sector) cannot be justified on efficiency grounds. Indeed, in so far as those requirements favour the customers who induce relatively higher provision costs, as it is the case by their same nature, they cannot be supported in the absence of redistributive preoccupations toward these individuals.

Observe that in Cremer *et Alii* [7], the equity concerns are addressed by assigning different weights in the welfare function to different economic agents. This is what we suggested above. These Authors show that, by proceeding like this, the outcome is the most efficient equilibrium which is feasible under the budget constraint. Analogous result would entail in our framework. Even more, similarly to what we said about the condition in (39), optimality would arise if it were not prevented by the presence of a binding budget constraint.

Yet attributing a weight larger than one to the utility components in the objective of the decision-maker sounds quite abstract and, in practice, it might prove hard to do so. An alternative option, often adopted in reality, is as follows. The social planner keeps on pursuing *utilitarian* welfare, that is her objective remains the same as in (22) and (36) for monopoly and duopoly respectively. However, hinging on the inner sphere of social judgments, she calibrates prices  $P^{s,r}$ ,  $\forall s$ , and frequency  $F^l$  according to the collective pro-islander bias<sup>39</sup>. For this policy to be more favourable to the islanders, as compared to  $(\mathbf{p}_I^{RM}, \mathbf{f}_I^{RM})$  and  $(\mathbf{p}_I^{PR}, \mathbf{f}_I^{PR})$  under monopoly and duopoly respectively, one should have  $F^l > f_I^{l,RM}$  and  $F^l > f_I^{l,PR}$ , together with  $P^{s,r} < p_I^{s,r,RM}$  and  $P^{s,r} < p_I^{s,r,PR}$ ; this is assumed to be the case.

Notice that the formulation we have adopted accommodates for the possibility that different tariffs be set in different seasons, though this does not need to happen. The decision actually depends on how concerned society is with islanders' welfare. Indeed, a uniform price  $P^r$  is rather imposed, if society also cares about smoothing the residents' pattern of expenses in shipping consumption all over the year. Conditionally on the chosen values, the other relevant variables can then be optimally characterized.

In formal terms, once the regulator commits to the initial decisions in favour of the islanders, such decisions enter the social problem as additional constraints. Therefore, it is a matter of solving the programme

$$\begin{aligned}
 & \underset{\{p_I^{s,i}, f_I^s\}_{\forall s,i}}{\text{Max}} \quad W \\
 & \text{subject to} \\
 & f_I^l = F^l \text{ and } p_I^{s,r} = P^{s,r}, \quad \forall s \\
 & \pi_I \geq 0.
 \end{aligned} \tag{45}$$

Observe that, since (45) incorporates the same objective function as (22) and (36) but

[21] multiplies the rent of the regulated supplier by a parameter  $\beta > 1$  in the objective function of a regulator who is captured by the industry, whereas the case of a regulator biased against the firm should be represented by assuming  $\beta < 1$ . Notice that a formulation of this kind, which attributes a weight to the firm's utility, would not suit our setup because, in a symmetric information world, the profits of the regulated firm are always driven to zero.

<sup>39</sup>We focus on the sole low-season frequency because, during the high season, the traffic is large enough to generate interesting business opportunities, so that public service obligations, beyond "standard" regulation, are probably unnecessary.

a wider set of constraints, the programme under-performs, as compared to (22) and (36). That is, the very structure of this programme involves that some efficiency be forgone for equity to be pursued, a pitfall which would be avoided if a weighed social welfare function were maximized<sup>40</sup>. The relevant variables, other than the committed ones, are determined according to (23a) and (23b), if a monopoly is regulated, and to (37a) and (37b), if a duopoly is partially regulated.

An appraisal is owed at this stage. In some European Member States, the price-cap methodology, even in the pure version without quality adjustments, is not yet applied to the shipping activities. For instance, according to the Italian Law 343/95, the tariffs of the services provided by the maritime companies that receive subsidies from the State are to be disciplined after the Law 856/86<sup>41</sup>. As a result, the services previously said are administratively deliberated by the Ministry of Economics and Finance, as supported by the NARS (Nucleo consulenza Attuazione linee guida Regolazione Servizi di pubblica utilità), together with the Ministry of Infrastructure and Transports. This circumstance has been generally perceived as a weakness of the wide regulatory reorganization process, which has concerned several other utilities during the last decade. The approach described above suggests that it might rather represent a feasible means to express a social preference, which would be hardly reflected otherwise. Nevertheless, it is difficult to rationalize why more rigorous procedures are neither adopted for the selection of the remaining relevant variables, except if one may responsibly claim that social preoccupations, other than efficiency, drive all such choices as well. And even in the latter case, a margin of improvement still exists.

The preference society expresses amounts to having the available market served, however thin it happens to be. For this universal service purpose, the regulated shipper is explicitly required to ensure the provision of a given number of connections even in the low season ( $F^l$ ), when traffic is essentially composed by residents and operation is hardly convenient<sup>42</sup>. It is as well obliged to charge the islanders with a price ( $P^{s,r}$ ) society judges to be sufficiently affordable, even if this renders the activity not self-financing.

Notice that securing frequency generates benefits also for the non-residents if, by any chance, they travel during the concerned season; however, this occurs to a limited extent and may not occur at all. Conversely, whatever the season, price bounds can be targeted solely to the residents. Altogether these circumstances involve that the burden associated with further scheduling is essentially borne by a restricted segment of the overall population of travellers. Let us try to understand how the story goes, in order to identify the concerned segment and the resulting implications from the firms' standpoint.

For the regulated firm's budget to be met, it is necessary to adjust the non-resident prices, that is cross-subsidization is called upon. Though this is the case under either industry structure, the (potential) presence of a second operator creates a difference between the two market scenarios, which should not be neglected.

In monopoly, there is no way the non-residents can avoid to fund the favourable conditions awarded to the residents because no outside option is available to them<sup>43</sup>.

In duopoly, to some extent, the unregulated shipper can attract non-residents by

<sup>40</sup> Technically speaking, instead of determining all the incumbent's tariffs and frequencies through a simultaneous optimization procedure, the regulator follows a multi-stage process. With a simultaneous procedure, all choice variables would be pinned down at once through a tatonnement process. In the case under scrutiny, the regulator initially fixes the redistribution conditions and then addresses the efficiency issues.

<sup>41</sup> The tariffs of the services of general interest, other than the ferry services, are generally subject to the rules established in Art. 10, Law 537/93, hence the price-cap methodology applies.

<sup>42</sup> The Convention signed for Corsica in 1976 by the French State with the maritime company SNCM compels the shipper to ensure seven daily ferry tours (plus the mixed cargo ones) during the winter. The number of tours the firm offers during the summer is, instead, much larger; it amounts to about 50.

<sup>43</sup> A natural outside option might be given by an alternative transportation mode. Nevertheless, in the present work, we restrain our attention to the ferry services and neglect the availability of other means.

(slightly) undercutting the incumbent during the high season. Interestingly, this provides a reason why the presence of a competitor, which is not compelled to social duties, can benefit some of those passengers: it offers them the possibility of partially escaping the subsidy they implicitly owe to the islanders.

On the opposite, the entrant is provided no incentive to supply a positive amount of service during the low season, that is entry is unlikely to occur during this period. Indeed, provided that rival frequencies are in a relationship of strategic substitutability, *ceteris paribus*, the larger the incumbent's number of connection, the smaller the competitor's. Of course, when the size of the traffic is little, as during the low season, this may well end up in the raise of a monopoly<sup>44</sup>. Then the markup shipper  $I$  is attributed on segment  $r$  is given by (see Appendix D for details)

$$\frac{p_I^{l,r} - (a + \gamma)}{p_I^{l,r}} = \frac{1}{\varepsilon_I^{(l,r)(l,r)}} \frac{\Gamma_p^{l,r,R} + \lambda^R}{1 + \lambda^R}. \quad (46a)$$

In (46a),  $\lambda^R$  is the Lagrange multiplier associated with the regulated shipper's budget constraint, the superscript  $R$  staying for *redistribution*. Moreover,  $\Gamma_p^{l,r,R} \equiv \lambda_p^{l,r,R} / X_I^{l,r}$ , which is defined as the ratio between the multiplier associated with the resident price constraint in season  $l$  and the aggregate resident demand, can be viewed as the shadow cost of the price obligation per ticket sold on segment  $r$ . Clearly, with  $\lambda_p^{l,r,R} > 0$ , the price in the right-hand side of (46a) is, in fact,  $P^{l,r}$ . Observe that, *ceteris paribus*, the larger the demand  $X_I^{l,r}$ , the smaller the unit shadow cost  $\Gamma_p^{l,r,R}$ , the smaller the shipper's markup. Intuitively, when consumption is important, the benefit to the islanders of a low tariff is big, so that it is socially worth sacrificing the provider's reward. While, in general, firms that operate both in monopolistic and in competitive markets (attempt to) partially finance their competitive activities by means of the monopolistic ones<sup>45</sup>, our analysis suggests that the imposition of PSOs for redistribution purposes may well revert the situation. Indeed, in our framework, the activities the regulated shipper performs as a monopolist during the low season are cross-subsidized by those it executes under more competitive conditions.

We shall finally concentrate on firm  $I$ 's scheduling for the case where it is a monopolist and only the islanders travel during the low season. Setting  $\tilde{\tau}^{l,r,R} \equiv \int \int \tau x_I^{l,r} g(\alpha, \tau) d\alpha d\tau$  and  $\hat{\tau}^{l,r,R} \equiv \int \int \tau \frac{\partial x_I^{l,r} / \partial p_I^{l,r}}{\partial X_I^{l,r} / \partial p_I^{l,r}} g(\alpha, \tau) d\alpha d\tau$ , the number of connections to be operated during the low season by shipper  $I$  writes as

$$f_I^{l,R} = \sqrt{\frac{\tilde{\tau}^{l,r,R} + (\lambda^R X_I^{l,r} + \lambda_p^{l,r,R}) \hat{\tau}^{l,r,R}}{2\phi_I (1 + \lambda^R) + 2\lambda_f^{l,R}}}, \quad (46b)$$

which equals  $F^l$  whenever  $\lambda_f^{R,l} > 0$ . As for the regulated monopoly, the amount of travels is the square root of a ratio which has a "weighed sum" of disutility of waiting time at the numerator and twice the marginal cost of connections at the denominator, the ratio being deflated to account for the multiplier of the budget constraint. Additionally the shadow costs of the constraints imposed to pursue redistribution show up in the environment

<sup>44</sup>In Cremer *et Alii* [7] things are different because, as already mentioned, qualities are strategic complements. Therefore, the imposition of a minimum standard, which directly affects the smaller quality, induces an indirect (strategic) increase in the larger quality. In our framework, imposing a lower bound on the incumbent's frequency (or the supply of a given number of connections, which is equivalent, as long as the constraint binds) reduces the rival's frequency and, eventually, drives it to zero. Then no entry occurs and firm  $I$  is a monopolist subject to PSOs.

<sup>45</sup>See, for instance, Calzolari and Scarpa [6].

here at stake. *Ceteris paribus*,  $f_I^{l,R}$  increases in the multiplier associated with the price constraint; this can be easily interpreted by considering that the larger the amount of connections to be offered, the more difficult to meet the price requirement. On the other hand,  $f_I^{l,R}$  decreases in the multiplier associated with the frequency constraint, meaning that the more severe the scheduling programme is, the more costly it is to be satisfied.

## 6.1 Discussion

The conclusion previously drawn as to cross-subsidization requires further qualification, as far as a duopolistic sector is at stake. At this aim, we hereafter rely on the result we summarized in our first Proposition, namely that, whenever two shippers are active on the market, people exhibiting low time value, those who are able to schedule their travels, behave as type-1 passengers and patronize the cheaper operator. On the other hand, people whose time value is relatively larger act as type 2 and take the first available ship.

As soon as partial regulation reflects the territorial continuity principle, the result previously recalled involves that the residents behave as type-1 passengers and patronize the regulated shipper, provided that the conditions secured in their interests are sufficiently favourable<sup>46</sup>. On the other side, as long as the wedge between rival prices is not too large, though type-1 non-residents tend to patronize the entrant, type-2 non-residents still randomize over the two shippers. Therefore, during the high season, either supplier serves a portion of such travellers, depending on the relative number of provided connections. As a matter of fact, diffuse evidence is found of such situations materializing in real-world shipping sectors: the non-residents manifest a certain tendency to allocate to the entrant, whereas the islanders generally patronize the regulated operator all over the year.

Overall, a few interesting conclusions can be derived, which we hereafter catalog.

1. In duopoly, the travellers who mainly bear the burden associated with the distributional concerns of society are not the non-residents as a whole, rather those such travellers who display particularly high disutility from waiting. This form of subsidization occurs *across market segments and seasons*.
2. Under the policy at stake, high- $\tau$  non-residents are required to provide implicit subsidy to the benefit of the residents even when the latter have equally high time value.
3. The presence of an unregulated shipper proves to be especially beneficial for type-1 non-residents, the ones who exhibit a limited degree of impatience; interestingly enough, this is the same as in the first-best environment we previously investigated, where pure efficiency were pursued.
4. In the high season, the residents can be asked to implicitly subsidize their same consumption in the low season, to the extent that the revenues collected on the islander segment during the high season contributes to cover the cost of the regulated service provision during the low season. In this perspective, subsidization occurs also *within market segment across seasons*. In duopoly, the subsidy involved is increasingly important, the more (type-1) non-residents patronize firm  $E$ , as this hardens the regulated operator's budget constraint<sup>47</sup>.

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<sup>46</sup>It is important to keep in mind that passengers' type allotment *endogenously* follows from the relationships between rival prices and frequencies. As previously said, the incumbent's offer is such that shipper  $E$  is crowded out during the low season and travellers are actually faced with a monopolist.

<sup>47</sup>The within-category effect can be expected to be sensitive to whether a uniform yearly price, which averages across low and high season, or different seasonal tariffs are charged.

5. In duopoly, the savings that become available to the low- $\tau$  non-residents, by travelling with the unregulated shipper, are seriously restrained by the strategic relationships existing between rival policies. The softer the competition the follower faces on the nonresidential market during the high season, the more significant the rent it enjoys.

Conclusion 5. deserves a few more words. As previously explained, given the social preferences, the partial regulator determines the incumbent's prices and frequencies so that she leaves appropriate room to shipper  $E$ . Therefore, conditionally on the need to discipline access and to ensure budget balance for the incumbent, the obtained solution is optimal, hence so is the associated rent. Yet, in a world where distributional concerns matter, giving up a net benefit to (part of) the industry is likely to raise a new delicate issue. In the following Section, we sketch a tentative discussion concerning the extraction of this rent.

## 6.2 The Unregulated Shipper's Rent: An Open Issue

If it were possible to transfer resources from industry to customers<sup>48</sup>, rather than across customers, then the pro-residents bias would *per se* work as a rent-extraction device. For this to occur, it would be necessary to ensure that rendering public service obligations more severe would cause a reduction in the profitability of the entrant's activity, other things being equal. However, except if subsidies can be attributed for uncovered costs, there is a limit to how heavy the incumbent's duties can be made. And even in the event that subventions are admitted, distorting taxation is then called for.

In the end, it is a matter of confronting the two following alternatives:

1. Allowing for passenger cross-subsidization and giving up a rent to the entrant.
2. Tightening the regulatory requirements to the (direct and/or indirect) benefit of *both* categories of customers, but increasing taxpayers' burden, and extracting the entrant's rent.

When option 1. prevails, one may still consider to pursue the rent-extraction objective by imposing a tax as a way of further tightening the regulatory requirements. However, taxation would be questionable for two main reasons: firstly, it might not be optimal; secondly, it is not sure that it represents a remedy as long as the maritime transport industry is concerned.

The choice of the appropriate tax would not be straightforward. For instance, it is not clear that it would pay to levy a tax on the level of sales<sup>49</sup>. To make sure that it would, one should be able to unambiguously assess the *economic* incidence of the tax. For imperfectly competitive sectors, this is generally a tricky task<sup>50</sup>. A preferable alternative would probably consist in a tax on economic profits. A proportional tax on the latter would change neither marginal cost nor marginal revenue. The targeted shipper would

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<sup>48</sup>As far as redistribution from the industry to passengers is concerned, resources could solely be taken away from the shipper which enjoys a net benefit, namely the entrant. Conversely, transfers from the regulated operator remain unfeasible, as long as the latter makes no profits.

<sup>49</sup>One may think about a tax conditioned on a sufficiently large traffic volume. This might amount to imposing a tax solely on the activity performed during the high season.

<sup>50</sup>The theory of tax incidence in oligopoly is poorly developed. A remarkable result is the one achieved by Delipalla and Keen [11], who show that, when the sales of an imperfectly competitive industry are subject to a tax, firms contract their outputs, but this is not necessarily detrimental to them. Of course, for any given level of before-tax profits, the providers are worse off, because they have to pay the tax; but as outputs are contracted, firms move closer to the cartel solution, hence their before-tax profits increase. Depending on how much outputs are cut back, it is theoretically possible for before-tax profits to increase by so much that suppliers are overall better off.

have no incentive to change its decisions about service provision and the prices paid by the passengers would not vary. To see this, suppose that the Government sets a tax rate  $t_\pi$  on economic profits. Then, shipper  $E$ 's objectives consists in maximizing its after-tax profits  $(1 - t_\pi) \pi_E$ . Clearly, whatever strategy maximizes before-tax profits  $\pi_E$  also maximizes after-tax profits  $(1 - t_\pi) \pi_E$ . It follows that the operator bears the whole tax burden and customers are not made worse off<sup>51</sup>.

Albeit a tax on profits would not distort choices at the margin, it should still be regarded with caution. Indeed, nowadays, the European fiscal climate is highly unattractive for ship-owners. This concern is seriously perceived at the European level, as it is evident from the following statement of the Commission communication C(2004) 43 drawing the Community guidelines on State aid to maritime transport [13]: "(...) many Member States have taken special measures to improve the fiscal climate for ship-owing companies". Therefore, adding further fiscal burden might prove in contradiction with the increasing adoption of support measures for maritime transport in Member States, especially as far as newly entered operators are concerned<sup>52</sup>.

After all, one should not be persuaded that the investigation about the appropriate regulation of the maritime ferry industry be exhausted once the (constrained) optimal pricing and scheduling are characterized and decentralizing devices are found. Redistribution considerations raise several surrounding issues, some of which remain open to debate. It is beyond the scope of the present work to convincingly develop all of those, however interesting they are. For the time being, we content ourselves with acknowledging the relevance of the ones we do not go through, as a first step toward the more comprehensive treatment they deserve.

## 7 Conclusions

How should maritime ferry industries be regulated? So far, this question has received no economically founded reply. Yet it raises an issue of fundamental importance, in so far as maritime transportation between islands and mainland is a service of general interest. As a matter of fact, it critically contributes to secure the national cohesion and integrity of countries which have islands and to promote the constitutionally recognized individual right to mobility (intended in a broad sense).

The question previously asked has been addressed in the present paper. We have argued that the appropriate design of regulatory policies crucially depends on whether society points to pure efficiency and/or to distributional objectives. Indeed, the efficient policy does not necessarily coincide with the one which guarantees effective territorial continuity and tutelage of the residents, the customers who are more seriously penalized by the drawbacks of insularity. Pursuing equity aims generally requires imposing PSOs beyond the regulatory duties efficiency calls for.

For the purpose of stylizing the peculiar features of the shipping sector, we have adopted a number of specific modelling devices. First of all, we have classified passengers into residents and non-residents, to whom different prices can be offered. Secondly, we have accounted for the significant traffic seasonality by identifying a high and a low season and allowing for a different amount of connections to be operated in each of those.

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<sup>51</sup>See Rosen [24] for a discussion on the matter.

<sup>52</sup>One such support measure is the flat rate tonnage taxation system ("tonnage tax"). According to the Commission communication C(2004) 43 [13], "'Tonnage tax' means that the shipowner pays an amount of tax linked directly to the tonnage operated (...) irrespective of the company's actual profits or losses". The tonnage tax entered into force first in Greece and was subsequently extended to several other States. Moreover, in the same communication, it is stated: "The Commission recognizes that launching short-sea shipping services may be accompanied by substantial financial difficulties which the Member States may wish to attenuate in order to ensure the promotion of such services". Short-sea shipping actually includes maritime ferry services such as the cabotage with the islands.

Under the previous circumstances, the prices charged on the two market segments are tied to finance the common cost of the provided travels representing the quality dimension of the service. Thirdly, we have restricted attention to the industry structures that are relevant in the European panorama, namely a monopoly and a duopoly, where the regulator imposes obligations which would not be assumed for pure commercial interests, as the EU Regulation 3577/92 [17] prescribes. Finally, in either regime, we have required that the regulated shipper's budget constraint be met in order to capture the European Commission's willingness to break too long a tradition of soft budgets and abusively diluted subsidies.

Within the framework recalled above, we have drawn and discussed a set of interesting results, some of which leading to new debatable subjects. To begin with, we have established some results which hold both in monopoly and in duopoly. Firstly, the optimal rule which governs the choice of each relevant variable (be it price or frequency) does not depend on the fact that other variables are simultaneously chosen. Moreover, because such variables relate to demand elasticities, they are contingent on the distribution of the individual characteristics, such as taste for the service and value of waiting time. Our findings reveal that this is so both when pure efficiency concerns are addressed and when the residents' welfare is assumed to be of particular concern for society. On the opposite, in the (ideal) first-best environment, prices exclusively reflect technological conditions. In turn, the pricing and scheduling PSOs, that are exogenously fixed to favour the islanders, solely embody the social value judgments.

We have as well drawn some conclusions which exclusively concern situations where entry matters. In particular, we have established that the presence of an unregulated shipper brings about advantages and create difficulties at once.

The rule which dictates how the regulated prices and frequencies should be optimally substituted, at the margin, becomes more complex when a second provider is active. Indeed, under duopoly, the regulated firm's viability needs be traded off against quite composite interests, those of the customers *and* of the rival operator. Instead, under monopoly, passengers are the only economic agents other than the shipper. Interestingly enough, the public sector has to make sure that the follower be not inefficiently crowded out. At the decentralization stage, this involves that the authority be able to use/elicit information about *both* the regulated and the unregulated shipper. By allocating a portion of the traffic to the entrant, the planner can be relatively less requiring *vis-à-vis* the regulated shipper.

At later stage, we have shed some light on the implications which follow, when the regulator puts forward the territorial continuity principle and addresses equity preoccupations by imposing PSOs on the incumbent. First of all, as these duties are particularly severe during the low season, when the traffic is essentially represented by islanders, the entrant's incentives to operate during this period are dampened. On the opposite, during the high season, budget requirements prevent the leader from vigorously competing on the nonresidential segment. Then soft competition shields activity profitability, so that the entrant is induced to provide its service by (slightly) undercutting the regulated leader.

As to the social aspects, our model predicts that a pro-residents planner needs to heavily rely on cross-subsidization and transfer the associated burden onto the nonresidential part of the population. More precisely, under duopoly, the burden is passed onto those non-residents who exhibit high value of time, hence large disutility from waiting. Importantly, this suggests that liberalization does not equally affect all customers. Potential beneficiaries appear to be the non-residents displaying relatively low penalty from waiting. The latter can (partially) escape the implicit subsidy owed to the islanders, in case the unregulated firm offers cheaper services. This is possible because they are sufficiently patient to wait for the ships of the unregulated firm and exclusively patronize this provider. Of course, no such outside option exists as long as the service is monopolistically supplied.

Yet one should be cautious about the savings that become available in duopoly. The latter are limited by the strategic complementarity between rival prices and, as a result, the entrant pockets a net rent. In a world where distribution matters, this last circumstance can be expected to raise new issues.

We would like to conclude with a few more points, which suggest directions of further research. Firstly, the whole analysis has been performed and the conclusions drawn under the (implicit) assumption that shippers charge linear prices. Nevertheless, in real-world ferry industries, frequent customers are usually offered the possibility of benefiting from quantity discounts, so that the unit price decreases as the number of purchased tickets gets larger. Formally speaking, this circumstance might be represented by allowing operators to propose two-part tariffs. Intuitively, the adoption of more sophisticated pricing instruments might induce a different allotment of passengers between providers. It would be interesting to explore this alternative environment.

Secondly, we have allowed for a single potential entrant. Yet we are not able to assess whether and to what extent this restriction affects the predictions of our model. In fact, this is a limit our analysis shares with several other works about access and competition in (partially) liberalized sectors. In their model about entry in postal markets, Cremer *et Alii* [8] have a similar word of caution on the matter.

Thirdly and lastly, we have characterized all regulatory policies in conditions of complete information. We acknowledge that this approach might not be fully convincing because, as it is documented, in transport industries informational asymmetries significantly beset the relationships between firms and authorities. Yet we would like to make an appraisal. For some scenarios, we have put forward a decentralization mechanism which has been shown to be little informationally demanding and, as such, implementable in practice (namely, the global price-and-frequency constraint *à la* De Fraja and Iozzi [10] in monopolistic sectors). More generally, we are persuaded that it was worth initiating the investigation of the regulatory framework in a frictionless scenario, to be perceived as a preliminary contribution to subsequent, more definitive, predictions.

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## APPENDICES

### A The Passengers' Endogenous Allotment

We start from the comparison between the  $(j, k)$ -option and the  $j$ -option. The  $\tau$ -passenger is better off by behaving as type 2, rather than patronizing firm  $j$ , whenever a relatively lower generalised cost is involved. The condition for this to be the case writes as

$$p^{s,e} + \frac{\tau}{2f^s} < p_j^s + \frac{\tau}{2f_j^s} \Leftrightarrow \tau > 2f_j^s (p_k^s - p_j^s). \quad (47a)$$

In the event that  $p_k^s > p_j^s$ , we can define  $\tau_{mg}^{s,2,j} \equiv 2f_j^s (p_k^s - p_j^s)$  the time value of the marginal customer<sup>53</sup>: people exhibiting larger  $\tau$  behave as type 2, whereas those with smaller  $\tau$  are better off by choosing firm  $j$ . In the opposite circumstance, that is with  $p_k^s < p_j^s$ , there does not exist  $\tau_{mg}^{s,2,j} > 0$ ; hence, all passengers prefer to act as type 2, rather than patronizing firm  $j$ .

Let us next compare the  $(j, k)$ -option with the  $k$ -option. The condition for the  $\tau$ -consumer to be type-2, instead of choosing firm  $k$ , is given by

$$p^{s,e} + \frac{\tau}{2f^s} < p_k^s + \frac{\tau}{2f_k^s} \Leftrightarrow \tau > 2f_k^s (p_j^s - p_k^s). \quad (47b)$$

With  $p_j^s > p_k^s$ , we can identify the cutoff time value  $\tau_{mg}^{s,2,k} \equiv 2f_k^s (p_j^s - p_k^s)$ , such that people with higher  $\tau$  act as type 2, those with lower  $\tau$  prefer travelling with firm  $k$  to being type 1. Conversely, with  $p_j^s < p_k^s$ , everybody is better off by using a unique aggregate service, rather than choosing always enterprise  $k$ . Remarkably, it is impossible that  $\tau_{mg}^{s,2,j}$  and  $\tau_{mg}^{s,2,k}$  simultaneously exist: whenever passengers split between patronizing firm  $j$ , say, and being type 2, nobody prefers firm  $k$  to acting as type 2. In the extreme event that  $p_j^s = p_k^s$ , we have  $\tau_{mg}^{s,2,k} = \tau_{mg}^{s,2,j} = 0$ , that is both cutoff values collapse onto the bottom of the support. In this scenario, those customers who suffer no disutility from waiting are indifferent between type-1 and type-2 behaviour, whereas all the others are better off by acting as type 2.

We finally compare the preference for firm  $j$  to that for firm  $k$ . The  $\tau$ -consumer is better off with the former if the associated generalised cost is relatively smaller, that is if

$$p_j^s + \frac{\tau}{2f_j^s} < p_k^s + \frac{\tau}{2f_k^s}.$$

Supposing, without loss of generality, that  $f_j^s > f_k^s$ , from the previous inequality we easily obtain

$$\tau > 2f_j^s f_k^s \left( \frac{p_j^s - p_k^s}{f_j^s - f_k^s} \right). \quad (49)$$

In the event that  $p_j^s > p_k^s$ , the time value which identifies the cutoff point over the support is given by  $\tau_{mg}^{s,1} \equiv 2f_j^s f_k^s (p_j^s - p_k^s) / (f_j^s - f_k^s)$ . Therefore, all customers with  $\tau > \tau_{mg}^{s,1}$  prefer enterprise  $j$  to  $k$ ; conversely, people with  $\tau < \tau_{mg}^{s,1}$  are better off with firm  $k$ . Notice that, under the previous assumption about frequencies, the condition on prices that is required for the existence of  $\tau_{mg}^{s,1}$  is the one under which  $\tau_{mg}^{s,2,j}$  does not exist, whereas  $\tau_{mg}^{s,2,k}$  does exist.

We are now equipped with all the information we need to identify the preference ordering structure; in what follows, we address this issue by describing passengers' behaviour

<sup>53</sup>This and all the other cutoff types we identify are indifferent between the two options they separate.

in each possible scenario, namely  $f_j^s > f_k^s$  together with  $p_j^s > p_k^s$  (Scenario 1) and  $f_j^s > f_k^s$  together with  $p_j^s < p_k^s$  (Scenario 2). Observe that we do not need to investigate also the case for  $f_j^s < f_k^s$ : this would provide no additional lesson, as results hold symmetrically.

### A.1 Scenario 1: $f_j^s > f_k^s$ and $p_j^s > p_k^s$

Whenever the operator charging higher price also provides larger frequency, the following outcomes are realized:

- $\exists \tau_{mg}^{s,1} > 0$ : Passengers with  $\tau > \tau_{mg}^{s,1}$  prefer firm  $j$  to firm  $k$ ; those with  $\tau < \tau_{mg}^{s,1}$  prefer firm  $k$  to firm  $j$ .
- $\nexists \tau_{mg}^{s,2,j} > 0$ : Whatever the time value, passengers prefer behaving as type 2 rather than patronizing operator  $j$ .
- $\exists \tau_{mg}^{s,2,k} > 0$ : Passengers with  $\tau > \tau_{mg}^{s,2,k}$  prefer acting as type 2 to choosing enterprise  $k$ ; those with  $\tau < \tau_{mg}^{s,2,k}$ , instead, prefer the  $k$ -option.

In order to relate the first point to the final one, we compare  $\tau_{mg}^{s,1}$  to  $\tau_{mg}^{s,2,k}$  and check whether any relation can be established between the two cutoff values. Indeed, it turns out that  $\tau_{mg}^{s,2,k} < \tau_{mg}^{s,1}$ . As a result, passengers' behaviour classifies as follows:

- Firm  $k$  is patronized by travellers whose  $\tau \in [0, \tau_{mg}^{s,2,k}]$ .
- The  $(j, k)$ -option prevails for travellers whose  $\tau \in (\tau_{mg}^{s,2,k}, +\infty)$ .

As it is evident,  $\tau_{mg}^{s,1}$  is irrelevant because travellers whose  $\tau \in (\tau_{mg}^{s,2,k}, \tau_{mg}^{s,1})$  prefer  $(j, k)$  to  $k$  and  $k$  to  $j$ .

### A.2 Scenario 2: $f_j^s > f_k^s$ and $p_j^s < p_k^s$

We now consider the case where the operator (here, firm  $j$ ) which offers the cheaper service also provides better quality. We have:

- $\nexists \tau_{mg}^{s,1} > 0$ : Whatever the time value, passengers prefer patronizing operator  $j$  rather than operator  $k$ .
- $\exists \tau_{mg}^{s,2,j} > 0$ : Passengers with  $\tau > \tau_{mg}^{s,2,j}$  are better off if they act as type 2 rather than waiting for firm  $j$ 's travels; the converse is true for those with  $\tau < \tau_{mg}^{s,2,j}$ .
- $\nexists \tau_{mg}^{s,2,k} > 0$ : Whatever the time value, passengers prefer behaving as type 2 rather than patronizing operator  $k$ .

Clearly, the only cutoff time value, which matters as to the classification of passengers' behaviour, is now  $\tau_{mg}^{s,2,j}$ ; hence, the following results are achieved:

- Firm  $j$  is patronized by travellers whose  $\tau \in [0, \tau_{mg}^{s,2,j}]$ .
- The  $(j, k)$ -option prevails for travellers whose  $\tau \in (\tau_{mg}^{s,2,j}, +\infty)$ .

## B The Regulated Monopoly Scheduling

The aim of this Appendix is to show how expression (26) in the text is found. We begin by rewriting the first-order condition with respect to  $f_I^s$  as

$$(1 + \lambda^{RM}) \sum_i \left[ p_I^{s,i} - (a + \gamma) \right] \frac{\partial X_I^{s,i}}{\partial f_I^s} + \frac{\partial V^s}{\partial f_I^s} = (1 + \lambda^{RM}) \phi_I.$$

The first order condition with respect to price yields the equality

$$p_I^{s,i} - (a + \gamma) = -\frac{\lambda^{RM}}{1 + \lambda^{RM}} \frac{X_I^{s,i}}{\partial X_I^{s,i} / \partial p_I^{s,i}}.$$

Replacing the latter condition into the former, we obtain

$$\lambda^{RM} \sum_i X_I^{s,i} \left( \frac{\partial X_I^{s,i} / \partial f_I^s}{-\partial X_I^{s,i} / \partial p_I^{s,i}} \right) + \frac{\partial V^s}{\partial f_I^s} = (1 + \lambda^{RM}) \phi_I. \quad (50)$$

Observe that, under monopoly, all passengers take a type-1 behaviour, so that one has

$$\begin{aligned} \frac{\partial V^s}{\partial f_I^s} &= \frac{1}{2 (f_I^s)^2} \int_{\tau} \int_{\alpha} \tau \sum_i x_I^{s,i} g(\alpha, \tau) d\alpha d\tau \\ &\equiv \frac{\tilde{\tau}^{s,RM}}{2 (f_I^s)^2}, \quad \forall s. \end{aligned}$$

Moreover, recalling (5a) and integrating both sides of the latter over the values of  $\tau$  and  $\alpha$  returns

$$\int_{\tau} \int_{\alpha} \frac{\partial x_I^{s,i}}{\partial f_I^s} g(\alpha, \tau) d\alpha d\tau = -\frac{1}{2 (f_I^s)^2} \int_{\tau} \int_{\alpha} \tau \frac{\partial x_I^{s,i}}{\partial p_I^{s,i}} g(\alpha, \tau) d\alpha d\tau,$$

where the left-hand side is equal to  $\partial X_I^{s,i} / \partial f_I^s$ .

Let us now replace the previous results into (50) in order to obtain

$$\lambda^{RM} \frac{1}{2 (f_I^s)^2} \sum_i X_I^{s,i} \frac{\int_{\tau} \int_{\alpha} \tau \frac{\partial x_I^{s,i}}{\partial p_I^{s,i}} g(\alpha, \tau) d\alpha d\tau}{\partial X_I^{s,i} / \partial p_I^{s,i}} + \frac{\tilde{\tau}^s}{2 (f_I^s)^2} = (1 + \lambda^{RM}) \phi_I.$$

Setting  $\int_{\tau} \int_{\alpha} \tau \frac{\partial x_I^{s,i} / \partial p_I^{s,i}}{\partial X_I^{s,i} / \partial p_I^{s,i}} g(\alpha, \tau) d\alpha d\tau \equiv \hat{\tau}^{s,i,RM}$ , the previous equality rewrites as

$$\left( \frac{1}{1 + \lambda^{RM}} \right) \left[ \lambda^{RM} \frac{1}{2 (f_I^s)^2} \sum_i X_I^{s,i} \hat{\tau}^{s,i,RM} + \frac{\tilde{\tau}^{s,RM}}{2 (f_I^s)^2} \right] = \phi_I, \quad \forall s.$$

Finally, rearranging terms, we obtain the expression in (26).

## C The Incumbent's Pricing Rule and Number of Connections under Partial Regulation

We begin by showing how one can find the optimal pricing condition in (40a). We shall subsequently provide details about the derivation of the number of connections in (40b).

## C.1 The Pricing Rule

We first rewrite (37a) as

$$(1 + \lambda^{PR}) \left[ p_I^{s,i} - (a + \gamma) \right] \frac{dX_I^{s,i}}{dp_I^{s,i}} = -\lambda^{PR} X_I^{s,i} - (p_E^{s,i} - a) \frac{\partial X_E^{s,i}}{\partial p_I^{s,i}}. \quad (51)$$

Using firm  $E$ 's first-order condition for pricing, we know that the margin  $(p_E^{s,i} - a)$  is equal to the ratio  $[-X_E^{s,i} / (\partial X_E^{s,i} / \partial p_E^{s,i})]$ . Replacing into (51) yields

$$(1 + \lambda^{PR}) \left[ p_I^{s,i} - (a + \gamma) \right] \frac{dX_I^{s,i}}{dp_I^{s,i}} = X_E^{s,i} \left( \frac{\partial X_E^{s,i} / \partial p_I^{s,i}}{\partial X_E^{s,i} / \partial p_E^{s,i}} \right) - \lambda^{PR} X_I^{s,i}$$

which, isolating firm  $I$ 's price margin on segment  $i$ , immediately becomes

$$p_I^{s,i} - (a + \gamma) = \frac{X_E^{s,i} \left( \frac{\partial X_E^{s,i} / \partial p_I^{s,i}}{\partial X_E^{s,i} / \partial p_E^{s,i}} \right) - \lambda^{PR} X_I^{s,i}}{\frac{dX_I^{s,i}}{dp_I^{s,i}}} \frac{1}{1 + \lambda^{PR}}.$$

Dividing both sides of the previous equality by  $p_I^{s,i}$  and manipulating terms returns

$$\frac{p_I^{s,i} - (a + \gamma)}{p_I^{s,i}} = \left( \frac{\lambda^{PR}}{-\frac{p_I^{s,i}}{X_I^{s,i}} \frac{dX_I^{s,i}}{dp_I^{s,i}}} + \frac{1}{-\frac{p_I^{s,i}}{X_I^{s,i}} \frac{dX_I^{s,i}}{dp_I^{s,i}}} \frac{p_E^{s,i}}{\partial p_I^{s,i}} \frac{\partial X_E^{s,i}}{\partial p_I^{s,i}} - \frac{1}{-\frac{p_E^{s,i}}{X_E^{s,i}} \frac{\partial X_E^{s,i}}{\partial p_E^{s,i}}} \right) \frac{1}{1 + \lambda^{PR}}.$$

We need now to remark, since cross price effects are symmetric, we have  $\partial X_E^{s,i} / \partial p_I^{s,i} = \partial X_I^{s,i} / \partial p_E^{s,i}$ . Defining  $\varepsilon_{IE}^{(s,i)(s,i)} \equiv (p_E^{s,i} / X_I^{s,i}) (\partial X_I^{s,i} / \partial p_E^{s,i})$  the cross price elasticity of demand  $X_I^{s,i}$  to price  $p_E^{s,i}$  and  $\hat{\varepsilon}_I^{(s,i)(s,i)} \equiv (p_I^{s,i} / X_I^{s,i}) (dX_I^{s,i} / dp_I^{s,i})$  the own price elasticity of demand  $X_I^{s,i}$  to price  $p_I^{s,i}$ , adjusted for the strategic interaction with the follower firm, it is straightforward to obtain

$$\frac{p_I^{s,i} - (a + \gamma)}{p_I^{s,i}} = \frac{\frac{\varepsilon_{IE}^{(s,i)(s,i)}}{\hat{\varepsilon}_I^{(s,i)(s,i)}} + \lambda^{PR}}{1 + \lambda^{PR}}, \quad \forall s, i,$$

which is the same as (40a) in the text.

## C.2 The Number of Connections

Let us rewrite (37b) as

$$-(1 + \lambda^{PR}) \left\{ \sum_i \left[ p_I^{s,i} - (a + \gamma) \right] \frac{dX_I^{s,i}}{df_I^s} - \phi_I \right\} = \frac{\partial V^s}{\partial f_I^s} + \sum_i X_E^{s,i} \left( \frac{\partial X_E^{s,i} / \partial f_I^s}{-\partial X_E^{s,i} / \partial p_E^{s,i}} \right), \quad \forall s.$$

Using the first-order condition with respect to price  $p_I^{s,i}$ , we further obtain

$$\begin{aligned} & \sum_i \left[ X_E^{s,i} \left( \frac{\partial X_E^{s,i} / \partial p_I^{s,i}}{\partial X_E^{s,i} / \partial p_E^{s,i}} \right) - \lambda^{PR} X_I^{s,i} \right] \left( \frac{dX_I^{s,i} / df_I^s}{dX_I^{s,i} / dp_I^{s,i}} \right) + \sum_i X_E^{s,i} \left( \frac{\partial X_E^{s,i} / \partial f_I^s}{-\partial X_E^{s,i} / \partial p_E^{s,i}} \right) \\ &= (1 + \lambda^{PR}) \phi_I - \frac{\partial V^s}{\partial f_I^s}, \quad \forall s. \end{aligned} \quad (52)$$

(??) can be manipulated to yield

$$\begin{aligned}
& \sum_i \left[ X_E^{s,i} \left( \frac{\partial X_E^{s,i}}{\partial p_E^{s,i}} \right) \left( \frac{dX_I^{s,i}}{dX_I^{s,i}} \right) - \lambda^{PR} X_I^{s,i} \left( \frac{dX_I^{s,i}}{dX_I^{s,i}} \right) \right] \\
& + \sum_i X_E^{s,i} \left( \frac{\partial X_E^{s,i}}{\partial p_E^{s,i}} \right) \\
& = (1 + \lambda^{PR}) \phi_I - \frac{\partial V^s}{\partial f_I^s}, \quad \forall s,
\end{aligned}$$

which is equivalent to

$$\begin{aligned}
& \sum_i \left[ \frac{1}{\frac{p_E^{s,i}}{X_E^{s,i}} \frac{\partial X_E^{s,i}}{\partial p_E^{s,i}}} \left( \frac{\partial X_I^{s,i}}{\partial p_E^{s,i}} \frac{p_E^{s,i}}{X_I^{s,i}} \right) - \frac{p_I^{s,i}}{\frac{p_I^{s,i}}{X_I^{s,i}} \frac{dX_I^{s,i}}{dp_I^{s,i}}} \frac{dX_I^{s,i}}{df_I^s} + \lambda^{PR} \frac{p_I^{s,i}}{\frac{p_I^{s,i}}{X_I^{s,i}} \frac{dX_I^{s,i}}{dp_I^{s,i}}} \frac{dX_I^{s,i}}{df_I^s} \right] \\
& + \sum_i \frac{p_E^{s,i}}{\frac{p_E^{s,i}}{X_E^{s,i}} \frac{\partial X_E^{s,i}}{\partial p_E^{s,i}}} \frac{\partial X_E^{s,i}}{\partial f_I^s} \\
& = (1 + \lambda^{PR}) \phi_I - \frac{\partial V^s}{\partial f_I^s}, \quad \forall s,
\end{aligned} \tag{53}$$

because  $\partial X_E^{s,i} / \partial p_I^{s,i} = \partial X_I^{s,i} / \partial p_E^{s,i}$  (cross-price effects are equal). Relying on our definitions of price elasticities, we can rewrite (53) as

$$\begin{aligned}
& \sum_i \left( \frac{\varepsilon_{IE}^{(s,i)(s,i)}}{\varepsilon_E^{(s,i)(s,i)}} \frac{p_I^{s,i}}{\hat{\varepsilon}_I^{(s,i)(s,i)}} \frac{dX_I^{s,i}}{df_I^s} + \lambda^{PR} \frac{p_I^{s,i}}{\hat{\varepsilon}_I^{(s,i)(s,i)}} \frac{dX_I^{s,i}}{df_I^s} \right) + \sum_i \frac{p_E^{s,i}}{\varepsilon_E^{(s,i)(s,i)}} \frac{\partial X_E^{s,i}}{\partial f_I^s} \\
& = (1 + \lambda^{PR}) \phi_I - \frac{\partial V^s}{\partial f_I^s}, \quad \forall s.
\end{aligned}$$

Manipulating further the previous condition, we find

$$\begin{aligned}
& \sum_i \left[ \frac{\varepsilon_{IE}^{(s,i)(s,i)}}{\varepsilon_E^{(s,i)(s,i)}} \frac{p_I^{s,i}}{\hat{\varepsilon}_I^{(s,i)(s,i)}} \left( \frac{dX_I^{s,i}}{df_I^s} \frac{f_I^s}{X_I^{s,i}} \right) \frac{X_I^{s,i}}{f_I^s} + \lambda^{PR} \frac{p_I^{s,i}}{\hat{\varepsilon}_I^{(s,i)(s,i)}} \left( \frac{dX_I^{s,i}}{df_I^s} \frac{f_I^s}{X_I^{s,i}} \right) \frac{X_I^{s,i}}{f_I^s} \right] \\
& - \sum_i \frac{p_E^{s,i}}{\varepsilon_E^{(s,i)(s,i)}} \left( - \frac{\partial X_E^{s,i}}{\partial f_I^s} \frac{f_I^s}{X_E^{s,i}} \right) \frac{X_E^{s,i}}{f_I^s} \\
& = (1 + \lambda^{PR}) \phi_I - \frac{\partial V^s}{\partial f_I^s}, \quad \forall s.
\end{aligned} \tag{54}$$

We can then use the definitions of frequency elasticities reported in the main text to rewrite (54) as

$$\begin{aligned}
& \sum_i \left( \frac{\varepsilon_{IE}^{(s,i)(s,i)}}{\varepsilon_E^{(s,i)(s,i)}} \frac{\hat{\eta}_I^{(s,i)(s)}}{\hat{\varepsilon}_I^{(s,i)(s,i)}} p_I^{s,i} \frac{X_I^{s,i}}{f_I^s} + \lambda^{PR} \frac{\hat{\eta}_I^{(s,i)(s)}}{\hat{\varepsilon}_I^{(s,i)(s,i)}} p_I^{s,i} \frac{X_I^{s,i}}{f_I^s} \right) - \sum_i \frac{\eta_{EI}^{(s,i)(s)}}{\varepsilon_E^{(s,i)(s,i)}} p_E^{s,i} \frac{X_E^{s,i}}{f_I^s} \\
& = (1 + \lambda^{PR}) \phi_I - \frac{\partial V^s}{\partial f_I^s}, \quad \forall s,
\end{aligned}$$

or, equivalently, as

$$\begin{aligned} & \sum_i \frac{\widehat{\eta}_I^{(s,i)(s)}}{\widehat{\varepsilon}_I^{(s,i)(s,i)}} R_I^{s,i} \left( \frac{\varepsilon_{IE}^{(s,i)(s,i)}}{\varepsilon_E^{(s,i)(s,i)}} + \lambda^{PR} \right) - \sum_i \frac{\eta_{EI}^{(s,i)(s)}}{\varepsilon_E^{(s,i)(s,i)}} R_E^{s,i} \\ &= f_I^s \left[ (1 + \lambda^{PR}) \phi_I - \frac{\partial V^s}{\partial f_I^s} \right], \quad \forall s. \end{aligned} \quad (55)$$

It is then straightforward to rearrange (55) at the aim of getting the expression for  $f_I^{s,PR}$  in (40b).

## D The Incumbent's Pricing and Frequency with Redistribution Concerns

In this Appendix, we show how conditions (46a) and (46b) in the text are obtained.

We consider the case where, in season  $l$ , firm  $I$  is a regulated monopolist. The condition for the (constrained) optimal choice of price  $p_I^{l,r}$ , which solves (45), writes as

$$(1 + \lambda^R) \left\{ \left[ p_I^{l,r} - (a + \gamma) \right] \frac{\partial X_I^{l,r}}{\partial p_I^{l,r}} + X_I^{l,r} \right\} + \frac{\partial V^{l,r}}{\partial p_I^{l,r}} + \lambda_p^{l,r,R} = 0,$$

where  $\lambda^R$  is the Lagrange multiplier associated with the budget constraint and  $\lambda_p^{l,r,R}$  the one of the price constraint. Recall that, by Roy's identity, it is  $\partial V^{l,r} / \partial p_I^{l,r} = -X_I^{l,r}$ . Therefore, the previous condition becomes

$$\lambda^R X_I^{l,r} + (1 + \lambda^R) \left[ p_I^{l,r} - (a + \gamma) \right] \frac{\partial X_I^{l,r}}{\partial p_I^{l,r}} + \lambda_p^{l,r,R} = 0.$$

Manipulating terms further yields

$$\frac{p_I^{l,r} - (a + \gamma)}{p_I^{l,r}} = \frac{1}{-\frac{\partial X_I^{l,r}}{\partial p_I^{l,r}} \frac{p_I^{l,r}}{X_I^{l,r}}} \frac{\frac{\lambda_p^{l,r,R}}{X_I^{l,r}} + \lambda^R}{1 + \lambda^R},$$

which is equivalent to

$$\frac{p_I^{l,r} - (a + \gamma)}{p_I^{l,r}} = \frac{1}{\varepsilon_I^{(l,r)(l,r)}} \frac{\frac{\lambda_p^{l,r,R}}{X_I^{l,r}} + \lambda^R}{1 + \lambda^R},$$

that is to (46a).

Let us now turn to the frequency in (46b), which is derived under the assumptions that firm  $I$  is a monopolist in season  $l$  and only residents actually travel during this period of the year. In this event, the condition for the (constrained) optimal choice of  $f_I^l$  writes as

$$(1 + \lambda^R) \left[ p_I^{l,r} - (a + \gamma) \right] \frac{\partial X_I^{l,r}}{\partial f_I^l} + \frac{\partial V^{l,r}}{\partial f_I^l} - \lambda_f^{l,R} = (1 + \lambda^R) \phi_I.$$

As already explained, under monopoly, any traveller behaves as type 1; this allows us to replace  $\partial X_I^{l,r} / \partial f_I^l$  with the ratio  $\widehat{\tau}^{l,r,R} / 2 (f_I^l)^2$ , where  $\widehat{\tau}^{l,r,R} \equiv - \int \int \tau \frac{\partial x_I^{l,r}}{\partial p_I^{l,r}} g(\alpha, \tau) d\alpha d\tau$ .

Similarly, we replace  $\partial V^{l,r} / \partial f_I^l$  with the ratio  $\widetilde{\tau}^{l,r,R} / 2 (f_I^l)^2$ , where we have  $\widetilde{\tau}^{l,r,R} \equiv$

$\int \int \tau x_I^{l,r} g(\alpha, \tau) d\alpha d\tau$ . Using the pricing condition as well, we obtain

$$\frac{\tilde{\tau}^{l,r,R}}{2(f_I^l)^2} + \frac{\hat{\tau}^{l,r,R}}{2(f_I^l)^2} \left( \lambda^R X_I^{l,r} + \lambda_p^{l,r,R} \right) - \lambda_f^{l,R} = (1 + \lambda^R) \phi_I,$$

which returns (46b) after few additional manipulations.